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## Interval type-2 control design for fuzzy-model-based systems

Li, Yuandi

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*A thesis submitted for the degree of  
Doctor of Philosophy*

# **Interval Type-2 Control Design for Fuzzy-Model-Based Systems**

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# Abstract

Fuzzy-model-based (FMB) control framework offers a systematic and effective approach for analyzing and synthesizing nonlinear dynamic systems. This thesis focuses on control design for FMB systems under interval type-2 (IT2) fuzzy logic. Both Takagi-Sugeno (T-S) FMB systems and polynomial fuzzy-model-based (PFMB) systems are investigated. IT2 fuzzy logic has been proposed to cope with the parameter uncertainties of the nonlinear systems. The main contribution of the thesis is presented in the following three parts:

In the first part, the problems of stabilization for IT2 T-S fuzzy systems with time-varying delay and parameter uncertainties are investigated. To facilitate the membership function dependent (MFD) stability analysis, piecewise linear membership functions have been employed to approximate the original upper and lower membership functions. More design flexibility and practicality could be achieved by imperfect premise matching, because it is not required that the fuzzy controller and fuzzy plant have the same premise membership function and/or number of fuzzy rules. The stability conditions are derived based on Lyapunov theory and are compared with condition based on membership function independent (MFI) approach.

In the second part, the problems of stabilization for IT2 T-S fuzzy systems with actuator saturation and parameter uncertainties are investigated. Following the first part, the information of the membership functions is included in the analysis. The actuator saturation is depicted and dealt with contractively invariant ellipsoid. The problem is formulated and solved with more flexibility due to imperfect premise matching.

In the third part, the problems of stabilization for IT2 polynomial fuzzy systems with time-varying delay and parameter uncertainties are investigated. The case has been extended from T-S FMB systems to PFMB systems compared to the first part. Because of the polynomial terms, the linear matrix inequality (LMI) approach used in the first part could not be conducted. The stability analysis is then investigated based on sum-of-squares (SOS) approach, which can be solved numerically by third-party toolbox.

Examples are presented to show the effectiveness of the proposed approach in the thesis.

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# Acronyms

FMB	Fuzzy-model-based
FOU	Footprint of uncertainty
IT2	Interval Type-2
LMI	Linear matrix inequality
MFD	Membership-function-dependent
MFI	Membership-function-independent
PDC	Parallel distributed compensation
PFMB	Polynomial fuzzy-model-based
SOS	Sum-of-squares
T-S	Takagi-Sugeno

# Chapter 1

## Introduction

### 1.1 Background

Intelligent control is the control methodology using various artificial computing approaches. And fuzzy control among other intelligent control sub-domains such as neural network control, machine learning control and genetic control is a control system based on fuzzy logic analyzing analog input values in terms of logical variables between 0 and 1.

Stability analysis and control synthesis for nonlinear systems has been a heated and challenging topic over the past few decades. This is because many real-world applications are nonlinear in nature while some nonlinear control techniques have their limitations, such as lack of rigorous stability analysis and applicable only to certain systems. Fuzzy control was then introduced as a systematic and effective control methodology to investigate the system stability and design the controller gain.

Although in the first place, fuzzy control was based on natural language and used as a model-free approach, it has successfully been applied to various industrial fields including cameras, vacuum cleaners, conditioners and so on [1]. At the late 1980s, model-based fuzzy control approach began to take the leading role in the theoretical field for rigorously analyzing system stability and performance. By converting the nonlinear model to the FMB one, one can then employ linear control techniques for the nonlinear systems, which is believed to be the advantage of fuzzy control.

In this thesis, system analysis and control synthesis of the FMB systems, especially the IT2 fuzzy systems are the main research issues. The main effort is to present stability conditions and design fuzzy controller for FMB systems with time-varying delays and actuator saturation. More details are presented in the following sections.

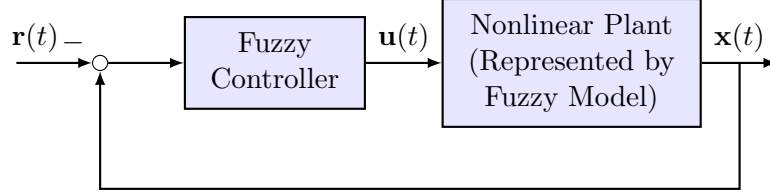


Figure 1.1: A block diagram of FMB control system.

## 1.2 Literature Review

FMB control framework has been successfully applied to control nonlinear systems in the past decades. In this section, related literature will be reviewed. In Section 1.2.1, basics of FMB systems are given, including T-S FMB systems and polynomial FMB systems. T-S FMB systems and polynomial FMB systems with IT2 fuzzy logic are also mentioned. In Section 1.2.2, sources of conservativeness of stability conditions and techniques used to reduce it are presented. In Section 1.2.3, FMB control systems with time-varying delay and actuator saturation are discussed as examples of extension of FMB control strategy. Finally, in Section 1.3, the objectives and organization of the thesis is demonstrated.

### 1.2.1 FMB Control System

For FMB control systems, stability analysis and control synthesis are very essential. Typically, the investigation of the stability of FMB control systems can be done along the following 5 steps [2]:

- 1) Represent the nonlinear plant with a fuzzy model.
- 2) Choose appropriate type of fuzzy controller for the control process.
- 3) Connect the fuzzy controller and fuzzy model to form a closed-loop FMB control systems.
- 4) Define a positive Lyapunov function.
- 5) Obtain the stability conditions based on Lyapunov stability theory.

A general structure of FMB control system is shown in Fig.1.1, which consists of a nonlinear plant represented by a fuzzy model and a fuzzy controller connected in a closed loop. The system state vector  $\mathbf{x}(t)$  combined with the input vector  $\mathbf{r}(t)$  will be processed by the fuzzy controller to generate the control signal  $\mathbf{u}(t)$ . The control signal is then input to the nonlinear plant for the control purposes.

This section will then continue to introduce some of the most important aspects of the FMB control systems. T-S FMB systems and LMIs based stability analysis will be first discussed. Polynomial FMB systems and SOS based stability analysis

will be then reviewed. After that, T-S/polynomial FMB systems with IT2 fuzzy logic will be introduced. At last, the parallel distributed compensation (PDC) and non-PDC [3–6] approach for the implementation of the control synthesis will be mentioned.

#### **1.2.1.1 T-S FMB System and LMIs based Stability Analysis**

The theory of T-S fuzzy model [7] has been developed for about 30 years to conduct system analysis and control synthesis for nonlinear systems. With the T-S fuzzy model, the nonlinear system dynamics can be represented as an average weighted sum of some local linear subsystems, where the weights are determined by membership functions. In this way, many linear control techniques can be applied to carry out stability analysis and control synthesis for FMB systems.

Based on the Lyapunov theory, an FMB control system is guaranteed to be asymptotically stable [8] if the time derivative of the Lyapunov function (non-negative function) is negative definite. One of the mathematical descriptions of the stability conditions is LMIs based conditions. For all the Lyapunov inequalities in terms of LMIs, if there exists a common solution, then the FMB control system is guaranteed to be asymptotically stable [8]. The search for the solution can be numerically solved by LMI toolbox in MATLAB. T-S FMB system has played an important role to conduct system analysis and control synthesis [9–13] for nonlinear systems in a systematic form for the last three decades.

#### **1.2.1.2 Polynomial FMB System and SOS based Stability Analysis**

Polynomial fuzzy-model-based (PFMB) system [14] was first proposed in 2009 to model and control nonlinear systems. It could be regarded as a powerful extension and generalization of the traditional T-S FMB systems, as polynomial terms are adopted in describing the dynamic of the nonlinear system in the consequent of the fuzzy rules and when the order of the polynomials is zero the PFMB system is reduced to traditional T-S FMB system.

Because of the polynomial terms, the original LMI based approach for stability analysis and control synthesis could not be conducted. The stability conditions for PFMB systems based on Lyapunov theory are then described in terms of SOS. Consequently, numerical results could be found with, for example, the third-party toolbox SOSTOOLS [15]. Some of the recent research on PFMB systems are listed as follows, for example, [16] studied stability analysis via approximated membership functions considering sector nonlinearity of control input, [17] studied stability analysis with mismatched premise membership functions and [18] studied stability analysis using switching polynomial Lyapunov function.

### 1.2.1.3 T-S/Polynomial MFB System with IT2 Fuzzy Logic

Type-1 fuzzy sets were first introduced in 1965 [19]. Type-2 fuzzy sets were then introduced in 1975 [20]. Since then, it has attracted great attention and many fruitful results have been presented in both theory and practice (see, e.g. [5, 21–26]). One motivation for studying such a class of systems is that type-2 fuzzy sets are better in representing and capturing uncertainties [27, 28], especially when the nonlinear plant inevitably suffers the parameter uncertainties while type-1 fuzzy sets do not contain uncertain information. Lots of successful applications can be found in the literature such as robot manipulators [29], face recognition [30], image processing [31], energy markets [32], linguistic summarization [33] and so on.

However, the general type-2 membership functions have huge complexities lying in the footprint of uncertainty (FOU) [34] which will lead to difficult analysis and high computational burden. Compromises have been made to adopt IT2 membership functions instead of general type-2 membership functions, where the membership grades of the secondary membership functions are constants rather than the functions of the premise variables.

By combining T-S/polynomial FMB systems with IT2 fuzzy logic, we can achieve IT2 T-S FMB/PFMB systems [4, 35–41]. This framework will represent the nonlinear dynamics with T-S FMB/PFMB systems and handle the uncertainties with IT2 fuzzy logic at the same time. In the thesis, the stability analysis and control synthesis of IT2 T-S/polynomial FMB system are the main research topics.

### 1.2.1.4 PDC and non-PDC Scheme for FMB Systems

The PDC scheme was first introduced in [8] suggesting that a linear sub-controller is designed to control each linear sub-system and the controller shares the same premise rules and membership functions as those of the fuzzy model. The PDC scheme will allow cross term of membership functions to be extracted in the stability analysis, which would lead to more relaxed stability conditions. However, the design flexibility of the controller may be lost. For example, if the membership functions of the fuzzy model are complicated or the number of the fuzzy rules of the fuzzy model is large, the fuzzy controller designed under PDC scheme will increase the implementation cost.

To overcome the drawbacks of the PDC scheme, imperfect premise matching [42] was proposed as one of the non-PDC schemes, where the fuzzy controller does not have to share the same number of rules and premise membership functions as those of the fuzzy model. By carefully choosing less complicated membership functions and fewer fuzzy rules for the controller, the implementation cost will be lower and more design flexibility can be achieved compared to the PDC scheme.

### 1.2.2 Relaxation of the Stability Conditions

For FMB control systems, the stability analysis remains as the most critical problem. While the conservativeness of the stability conditions can result in infeasible solutions even though the system is controllable. Therefore, the applicability of FMB control framework is limited. To better relax the stability conditions, i.e. reduce the conservativeness, three main sources of conservativeness are discussed in the following.

#### 1.2.2.1 Dimensions of Fuzzy Summation

For FMB control systems, how to deal with the fuzzy summation of the grades of membership functions is the first source of conservativeness. Under MFI analysis approach together with PDC scheme, the terms of the fuzzy summation corresponding to the same membership functions can be collected to produce less conservative stability conditions. The works in [43] extended the fuzzy summation to higher dimension to relax the stability conditions. Further on, the above works were generalized in [44] by applying Pólya's permutation theory [45] providing asymptotically necessary and sufficient conditions. Since then, Pólya's theory has been applied to expand the degree of fuzzy summation to further relax the stability conditions.

#### 1.2.2.2 Types of Lyapunov Functions

In the study of the stability analysis of FMB systems, the different types of Lyapunov functions affect the conservativeness as well. The quadratic Lyapunov function with its first order derivative are commonly investigated in the stability analysis [8]. To further relax the stability conditions, more general types of Lyapunov functions have been exploited, such as piecewise linear Lyapunov function [46], switching Lyapunov function [47], fuzzy Lyapunov function [48, 49] and polynomial Lyapunov function [47].

#### 1.2.2.3 Information of Membership Functions

MFD stability analysis, which makes use of the information of membership functions demonstrates a greater potential than the MFI one for relaxing the conservativeness of stability results [50]. For MFD stability conditions, the information of both the fuzzy model and controller is taken into account in the stability analysis. The information of membership functions are added to the stability conditions by slack matrices through S-procedure [51]. Consequently, the number of stability conditions will be higher and the computational demand to solve the stability conditions will be higher too. Since the membership function information are concerned in MFD analysis, the stability conditions obtained are no longer for any shape of membership functions but only valid for the FMB systems to be controlled.



To include the information of the membership functions, the boundary information, i.e. the lower and/or upper bounds of the global operating domain can be applied in the stability analysis [42, 52, 53]. The largest and smallest grades of membership are considered as the upper and lower bounds and will be used in the stability analysis. Later on, regional information of membership functions have been applied to stability analysis. The idea is to divide the membership functions into sub-regions, and in each sub-region, local upper and lower bounds of the grade of membership functions will be used in the stability analysis. More relaxed stability conditions can be achieved [54, 55] by using the local membership functions boundary information.

However, when the membership functions are brought to the stability analysis, the number of stability conditions will become infinity. To overcome this difficulty, membership function approximation was proposed. Different forms of membership functions such as staircase [56, 57], piecewise linear [58] and polynomial [59, 60] membership functions are used to approximate the original membership functions. Because the grade of membership for these approximated membership functions can be gained based on sample points, the number of stability conditions will no longer be infinity and the solutions can be found by convex programming techniques.

### **1.2.3 Extension of FMB Control Strategy**

FMB control strategy has been applied to various nonlinear control problems, for example, the systems with time-varying delay [61–68] and systems with actuator saturation [69–71]. In the thesis, FMB systems with time-varying delay and FMB systems with actuator saturation will be investigated.

#### **1.2.3.1 FMB Systems with Time-varying Delay**

It is well known that most practical dynamic systems inherently involve time delays. Without taking the limitations into consideration, techniques developed may result in performance degradation or even instability of the closed-loop control system in practice. In recent years, FMB system with time delays has been probed widely through delay-independent approach [61–63] and delay-dependent approach [64–66]. The former approach (delay-independent approach) does not involve time delay into the analysis, and the resulting stable FMB system would remain stable for other values of the delay. While the latter approach (delay-dependent approach) normally considers the information of the delay, and various inequalities would be applied to approximate the bound of the delay-related terms like the delay itself and its derivative. Less conservative result is expected as more information of the delay is involved in delay-dependent approach. Just to name a few more recent results for delay-dependent approach, in [67], the authors used delay partitioning approach to reduce the conservatism of delayed T-S fuzzy systems. In [68], the authors dealt

with the delay with input-output approach and two-term approximation where time-varying delay was treated as a kind of uncertainty to design filter. These methods could be even combined to get less conservative results. Although we get plenty of meaningful results on FMB systems with delays, there is still room left for us to make further extension, especially for IT2 T-S FMB and PMFB control systems.

### 1.2.3.2 FMB Systems with Actuator Saturation

Most practical dynamic systems involve actuator saturation as physical capacity of the actuator is always limited. Without taking the limitations into consideration, techniques developed may result in severe performance degradation or even instability of the closed-loop control system. In recent years, fuzzy system with actuator saturation has been probed widely. Just to name a few, in [69], the authors proposed robust stability analysis and fuzzy-scheduling control for nonlinear system by maximizing the domain of attraction of a T-S fuzzy system. In [70], the authors investigated  $H_\infty$  control problem subject to actuator saturation for nonlinear systems by the fuzzy scheme. In [71], the authors dealt with performance constrained control problem for nonlinear stochastic systems subject to  $H_\infty$  performance constraint and actuator saturation.

## 1.3 Objectives and Organization

The main objective of the thesis is to relax the stability conditions of FMB control systems using membership function information resulting in improving the applicability of FMB control strategy. State-feedback IT2 fuzzy controllers are designed for IT2 T-S/polynomial FMB systems under imperfect premise matching (the controller does not have to share the same premise membership functions and/or number of fuzzy rules with the plant). The details are as follows:

- 1) Develop relaxed stability conditions with IT2 fuzzy controllers for T-S FMB systems with time-varying delay.
- 2) Develop relaxed stability conditions with IT2 fuzzy controllers for T-S FMB systems with actuator saturation.
- 3) Develop relaxed stability conditions with IT2 polynomial fuzzy controllers for PFMB systems with time-varying delay.

Based on the above objectives, the thesis can be separated into three main chapters as shown in Fig.1.2. The relaxation of the stability conditions are discussed in these chapters using piecewise linear membership function approximation approach. The remainder of the thesis is organized as follows:

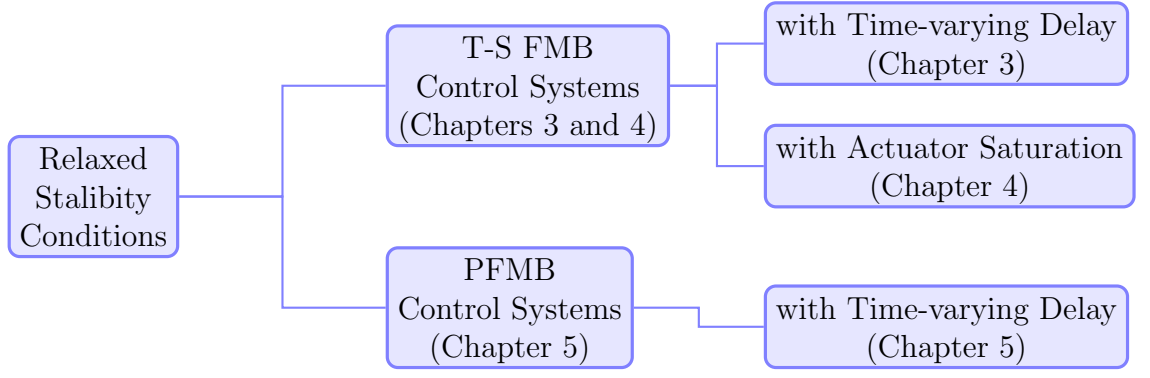


Figure 1.2: Relation of main chapters.

- In Chapter 2, the preliminaries of IT2 T-S FMB and PFMB control systems are presented, including notations, model construction, useful lemmas, etc. These are the background knowledge for the subsequent chapters
- In Chapter 3, stability conditions for IT2 T-S FMB control systems with time-varying delay are investigated. The stability conditions are summarized in terms of LMIs. The information of the IT2 membership functions are utilized to relax the stability conditions.
- In Chapter 4, stability conditions for IT2 T-S FMB control systems with actuator saturation are investigated. The stability conditions are summarized in terms of SOS. The information of the IT2 membership functions are utilized to relax the stability conditions.
- In Chapter 5, stability conditions for IT2 PFMB control systems with time-varying delay are investigated. The stability conditions are summarized in terms of SOS. The information of the IT2 membership functions are utilized to relax the stability conditions. This chapter can be regarded as the generalization of Chapter 3 from T-S FMB systems to PFMB systems with time-varying delay.
- In Chapter 6, conclusion of the thesis is drawn together with the future work plan.

# Chapter 2

## Preliminaries

In this chapter, the preliminaries which support the research work in the thesis will be introduced. Firstly, the notations adopted in the thesis are given. Secondly, sector nonlinearity technique for construction of the FMB systems is briefly introduced and IT2 membership functions are discussed. Thirdly, the IT2 T-S FMB control system and IT2 PFMB control system are presented respectively. Lastly, some useful lemmas used in the thesis are presented.

### 2.1 Notation

The notation in the thesis is quite standard. The expressions of  $\mathbf{M} > 0$ ,  $\mathbf{M} \geq 0$ ,  $\mathbf{M} < 0$  and  $\mathbf{M} \leq 0$  denote the positive, semi-positive, negative and semi-negative definite matrices  $\mathbf{M}$ , respectively.  $\mathbf{M} > \mathbf{N}$  means  $\mathbf{M} - \mathbf{N} > 0$ . The symbol “\*” in a matrix represents the transposed element in the corresponding position. The symbol “diag{...}” stands for a block-diagonal matrix. The superscript “ $T$ ” represents the transpose. The superscript “ $-1$ ” represents the inverse. The following notation is employed [72]. A monomial in  $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$  is a function of the form  $x_1^{d_1}(t)x_2^{d_2}(t)\cdots x_n^{d_n}(t)$ , where  $d_i \geq 0, i = 1, 2, \dots, n$ , are integers. The degree of a monomial is  $d = \sum_{i=1}^n d_i$ . A polynomial  $\mathbf{p}(\mathbf{x}(t))$  is a finite linear combination of monomials with real coefficients. A polynomial  $\mathbf{p}(\mathbf{x}(t))$  is an SOS if it can be written as  $\mathbf{p}(\mathbf{x}(t)) = \sum_{j=1}^m \mathbf{q}_j(\mathbf{x}(t))^2$ , where  $\mathbf{q}_j(\mathbf{x}(t))$  is a polynomial and  $m$  is a nonnegative integer. It can be concluded that if  $\mathbf{p}(\mathbf{x}(t))$  is an SOS, then  $\mathbf{p}(\mathbf{x}(t)) \geq 0$ .

The mathematical fonts are in standard format: scalars are in italic fonts; vectors are in bold fonts; and matrices are in bold and capital fonts.

### 2.2 Sector Nonlinearity Technique

There are general two approaches [73] to construct the nonlinear systems to fuzzy model. One is through system identification using the input-output data, the other is using derivation of the system dynamic equations. In the thesis, the sector non-

linearity technique [8, 74] from the latter approach will be adopted to represent the nonlinearity terms and then construct the fuzzy model.

The aim of sector nonlinearity technique is to find global sector such that the nonlinear terms can be represented as the weighted combination of two linear terms, where the weights are membership functions in the fuzzy model.

For example (time  $t$  is omitted in this example), if the nonlinear term is chosen to be  $f_1(\mathbf{x})$ , we have the upper and lower bounds  $f_{1_{max}}, f_{1_{min}}$ . Conceptually, we use following fuzzy rules to interpret the modeling process:

Rule 1 : IF  $f_1(\mathbf{x})$  is around  $f_{1_{min}}$ ,

THEN  $f_1(\mathbf{x}) = f_{1_{min}}$ ;

Rule 2 : IF  $f_1(\mathbf{x})$  is around  $f_{1_{max}}$ ,

THEN  $f_1(\mathbf{x}) = f_{1_{max}}$ .

The membership functions are exploited to combine the fuzzy rules. To calculate the grades of membership, we employ the following relations:

$$\begin{aligned} f_1(\mathbf{x}) &= \mu_{M_1^1}(\mathbf{x})f_{1_{min}} + \mu_{M_1^2}(\mathbf{x})f_{1_{max}}, \\ \mu_{M_1^1}(\mathbf{x}) + \mu_{M_1^2}(\mathbf{x}) &= 1, \end{aligned}$$

where  $\mu_{M_1^1}(\mathbf{x})$  and  $\mu_{M_1^2}(\mathbf{x})$  are the grades of membership corresponding to the fuzzy terms  $M_1^1$  and  $M_1^2$ , respectively. In this case, the fuzzy terms  $M_1^1$  and  $M_1^2$  are “around  $f_{1_{min}}$ ” and “around  $f_{1_{max}}$ ”, respectively. Therefore, we can obtain

$$\mu_{M_1^1}(\mathbf{x}) = \frac{f_1(\mathbf{x}) - f_{1_{max}}}{f_{1_{min}} - f_{1_{max}}}, \mu_{M_1^2}(\mathbf{x}) = 1 - \mu_{M_1^1}(\mathbf{x}).$$

The general nonlinear system investigated in this thesis is the autonomous input-affine system in the following state-space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t), \quad (2.1)$$

where  $t$  is the continuous time in seconds;  $\mathbf{x}(t) \in \mathbb{R}^n$  is the system state vector;  $\mathbf{A}(\mathbf{x}(t)) \in \mathbb{R}^{n \times n}$  is the system matrix;  $\mathbf{B}(\mathbf{x}(t)) \in \mathbb{R}^{n \times l}$  is the input matrix; and  $\mathbf{u}(t) \in \mathbb{R}^l$  is control input. The fuzzy modeling process in this thesis is achieved by sector nonlinearity technique to represent each nonlinear term in  $\mathbf{A}(\mathbf{x}(t))$  and  $\mathbf{B}(\mathbf{x}(t))$ . For the polynomial fuzzy model, the Taylor series expansion [75], which is an extension to sector nonlinearity technique, is employed to represent the nonlinear terms as polynomial terms. Since this more advanced technique is not directly applied in the thesis, the technical details are omitted.

### 2.2.1 IT2 Membership Functions

Considering the embedded uncertainty of the nonlinear systems, the grade of the membership function will become an interval value. Following the previous discussion, it is known that:

$$\mu_{M_1^1}(\mathbf{x}) = \frac{f_1(\mathbf{x}) - f_{1_{max}}}{f_{1_{min}} - f_{1_{max}}}, \mu_{M_1^2}(\mathbf{x}) = 1 - \mu_{M_1^1}(\mathbf{x}).$$

When the nonlinear term  $f_1(\mathbf{x})$  has uncertainty making  $f_1(\mathbf{x}) \in [f_1^L(\mathbf{x}), f_1^U(\mathbf{x})]$ , the above  $\mu_{M_1^1}(\mathbf{x})$  and  $\mu_{M_1^2}(\mathbf{x})$  will be rendered into interval sets  $\tilde{\mu}_{M_1^1}(\mathbf{x})$  and  $\tilde{\mu}_{M_1^2}(\mathbf{x})$  as follows:

$$\begin{aligned} \tilde{\mu}_{M_1^1}(\mathbf{x}) &\in [\underline{\mu}_{M_1^1}(\mathbf{x}), \bar{\mu}_{M_1^1}(\mathbf{x})], \\ \tilde{\mu}_{M_1^2}(\mathbf{x}) &\in [\underline{\mu}_{M_1^2}(\mathbf{x}), \bar{\mu}_{M_1^2}(\mathbf{x})]. \end{aligned}$$

Combining the IT2 membership function and the fuzzy model, the uncertainty and nonlinearity of the nonlinear systems can be captured in IT2 T-S/polynomial FMB system.

## 2.3 IT2 T-S FMB Control System

### 2.3.1 IT2 T-S Fuzzy Model

Consider a nonlinear system with parameter uncertainties represented by the following IT2 T-S fuzzy model with lower and upper membership functions.

**Plant Rule  $i$ :**

IF  $\theta_1(\mathbf{x}(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(\mathbf{x}(t))$  is  $\tilde{M}_{i2} \cdots$  and  $\theta_\Psi(\mathbf{x}(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t), \quad (2.2)$$

where  $\tilde{M}_{i\alpha}$  is an IT2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector,  $\mathbf{u}(t) \in \mathbb{R}^l$  is the input vector.  $\mathbf{A}_i \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B}_i \in \mathbb{R}^{n \times l}$  are known matrices as system matrices and input matrices, respectively. The firing strength of rule  $i$  is the interval sets as follows:

$$W_i(\mathbf{x}(t)) = [\underline{w}_i(\mathbf{x}(t)), \bar{w}_i(\mathbf{x}(t))], \quad i = 1, 2, \dots, p, \quad (2.3)$$

where

$$\underline{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq 0, \quad (2.4)$$

$$\bar{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq 0, \quad (2.5)$$

in which  $\underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t)))$  and  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t)))$  denote the lower and upper membership functions, respectively, satisfying the property  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq 0$ , and  $\underline{w}_i(\mathbf{x}(t))$  and  $\bar{w}_i(\mathbf{x}(t))$  denote the lower and upper grade of membership respectively. The inferred IT2 T-S fuzzy model is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)), \quad (2.6)$$

where

$$\tilde{w}_i(\mathbf{x}(t)) = \underline{w}_i(\mathbf{x}(t))\underline{\alpha}_i(\mathbf{x}(t)) + \bar{w}_i(\mathbf{x}(t))\bar{\alpha}_i(\mathbf{x}(t)) \geq 0, \quad \forall i, \quad (2.7)$$

$$\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1, \quad (2.8)$$

in which  $\underline{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(\mathbf{x}(t)) + \bar{\alpha}_i(\mathbf{x}(t)) = 1$ .

### 2.3.2 IT2 T-S Fuzzy Controller

An IT2 T-S fuzzy controller of  $c$  rules is employed to control the nonlinear plant subject to parameter uncertainties represented by the IT2 T-S fuzzy model (2.6), where the  $j$ -th control rule is of the following format:

**Controller Rule  $j$ :**

IF  $\sigma_1(\mathbf{x}(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(\mathbf{x}(t))$  is  $\tilde{N}_{j2} \cdots$  and  $\sigma_\Omega(\mathbf{x}(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$\mathbf{u}(t) = \mathbf{K}_j\mathbf{x}(t), \quad (2.9)$$

where  $\tilde{N}_{j\beta}$  is an IT2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ ,  $\mathbf{K}_j$  are the feedback gains to be determined. The firing strength of rule  $j$  is the interval sets as follows:

$$M_j(\mathbf{x}(t)) = [\underline{m}_j(\mathbf{x}(t)), \bar{m}_j(\mathbf{x}(t))], \quad j = 1, 2, \dots, c, \quad (2.10)$$

where

$$\underline{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq 0, \quad (2.11)$$

$$\bar{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq 0, \quad (2.12)$$

in which  $\underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t)))$  and  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t)))$  denote the lower and upper membership functions respectively satisfying the property  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq 0$ , and  $\underline{m}_j(\mathbf{x}(t))$  and  $\bar{m}_j(\mathbf{x}(t))$  denote the lower and upper grade of membership

respectively. The inferred IT2 T-S fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) \mathbf{K}_j \mathbf{x}(t), \quad (2.13)$$

where

$$\begin{aligned} \tilde{m}_j(\mathbf{x}(t)) &= \frac{\underline{m}_j(\mathbf{x}(t)) \underline{\beta}_j(\mathbf{x}(t)) + \overline{m}_j(\mathbf{x}(t)) \overline{\beta}_j(\mathbf{x}(t))}{\sum_{k=1}^c \left( \underline{m}_k(\mathbf{x}(t)) \underline{\beta}_k(\mathbf{x}(t)) + \overline{m}_k(\mathbf{x}(t)) \overline{\beta}_k(\mathbf{x}(t)) \right)} \\ &\geq 0, \quad \forall j, \end{aligned} \quad (2.14)$$

$$\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1, \quad (2.15)$$

in which  $\underline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$ ,  $\overline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(\mathbf{x}(t)) + \overline{\beta}_j(\mathbf{x}(t)) = 1$ .

### 2.3.3 IT2 T-S FMB Control System

With the plant and controller expressions in (2.6) and (2.13), and the property of  $\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1$ ,  $\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1$ ,  $\sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) = 1$ , we can have the IT2 T-S FMB control systems as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t)) ((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t)), \quad (2.16)$$

where  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t))$ .

## 2.4 IT2 PFMB Control System

### 2.4.1 IT2 Polynomial Fuzzy Model

Consider a nonlinear system with parameter uncertainties represented by the following IT2 polynomial fuzzy model with lower and upper membership functions.

**Plant Rule  $i$ :**

IF  $\theta_1(\mathbf{x}(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(\mathbf{x}(t))$  is  $\tilde{M}_{i2} \cdots$  and  $\theta_\Psi(\mathbf{x}(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i(\mathbf{x}(t)) \mathbf{x}(t) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{u}(t), \quad (2.17)$$

where  $\tilde{M}_{i\alpha}$  is an IT2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector,  $\mathbf{u}(t) \in \mathbb{R}^l$  is the input vector.  $\mathbf{A}_i(\mathbf{x}(t)) \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B}_i(\mathbf{x}(t)) \in \mathbb{R}^{n \times l}$  are known polynomial matrices as system matrices and input matrices, respectively. The firing strength of rule  $i$  is the interval sets as (2.3) where (2.4) and (2.5) hold,



in which  $\underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t)))$  and  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t)))$  denote the lower and upper membership functions, respectively, satisfying the property  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(\mathbf{x}(t))) \geq 0$ , and  $\underline{w}_i(\mathbf{x}(t))$  and  $\bar{w}_i(\mathbf{x}(t))$  denote the lower and upper grade of membership respectively. The inferred IT2 polynomial fuzzy model is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t))(\mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{u}(t)), \quad (2.18)$$

where (2.7) and (2.8) hold, in which  $\underline{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(\mathbf{x}(t)) + \bar{\alpha}_i(\mathbf{x}(t)) = 1$ .

## 2.4.2 IT2 Polynomial Fuzzy Controller

An IT2 polynomial fuzzy controller of  $c$  rules is employed to control the nonlinear plant subject to parameter uncertainties represented by the IT2 polynomial fuzzy model (2.18), where the  $j$ -th control rule is of the following format:

**Controller Rule  $j$ :**

IF  $\sigma_1(\mathbf{x}(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(\mathbf{x}(t))$  is  $\tilde{N}_{j2} \cdots$  and  $\sigma_\Omega(\mathbf{x}(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$\mathbf{u}(t) = \mathbf{K}_j(\mathbf{x}(t))\mathbf{x}(t), \quad (2.19)$$

where  $\tilde{N}_{j\beta}$  is an IT2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ ,  $\mathbf{K}_j(\mathbf{x}(t))$  are the polynomial feedback gains to be determined. The firing strength of rule  $j$  is the interval sets as (2.10) where (2.11) and (2.12) hold, in which  $\underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t)))$  and  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t)))$  denote the lower and upper membership functions respectively satisfying the property  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(\mathbf{x}(t))) \geq 0$ , and  $\underline{m}_j(\mathbf{x}(t))$  and  $\bar{m}_j(\mathbf{x}(t))$  denote the lower and upper grade of membership respectively. The inferred IT2 polynomial fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t))\mathbf{x}(t), \quad (2.20)$$

where (2.14) and (2.15) hold, in which  $\underline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\beta}_j(\mathbf{x}(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(\mathbf{x}(t)) + \bar{\beta}_j(\mathbf{x}(t)) = 1$ .

## 2.4.3 IT2 PMFB Control System

With the plant and controller expressions in (2.18) and (2.20), and the property of  $\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1$ ,  $\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1$ ,  $\sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t)) = 1$ , we can have the IT2 PFMB control systems as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t))((\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t)))\mathbf{x}(t)), \quad (2.21)$$

where  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t))$ .

## 2.5 Useful Lemmas

The following lemmas are employed in the following chapters.

**Lemma 1 (Schur complement)** [51] *With matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  of appropriate dimensions and  $\mathbf{A} = \mathbf{A}^T$ ,  $\mathbf{C} = \mathbf{C}^T$ , the following relation holds:*

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{bmatrix} > 0. \iff \mathbf{C} > 0, \mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T > 0.$$

**Lemma 2 (S-procedure)** [51] *With symmetric matrices  $\mathbf{T}_0, \dots, \mathbf{T}_p$  and vector  $\mathbf{v}$  of appropriate dimensions, the following relation holds:*

$$\begin{aligned} & \text{there exists } \tau_1 \geq 0, \dots, \tau_p \geq 0 \text{ such that } \mathbf{T}_0 - \sum_{i=1}^p \tau_i \mathbf{T}_i > 0. \\ \implies & \mathbf{v}^T \mathbf{T}_0 \mathbf{v} > 0 \text{ holds for all } \mathbf{v} \neq \mathbf{0} \text{ that satisfy } \mathbf{v}^T \mathbf{T}_i \mathbf{v} > 0, i = 1, 2, \dots, p. \end{aligned}$$

**Lemma 3 (Jensen's inequality)** [76] *With  $\mathbf{x}(t)$ ,  $\mathbf{Q}$  of appropriate dimension,  $\mathbf{Q} > 0$  and  $h > 0$ , the following inequality holds :*

$$\begin{aligned} & -h \int_{t-h}^t \dot{\mathbf{x}}(\varphi)^T \mathbf{Q} \dot{\mathbf{x}}(\varphi) d\varphi \\ & \leq -(\mathbf{x}(t) - \mathbf{x}(t-h))^T \mathbf{Q} (\mathbf{x}(t) - \mathbf{x}(t-h)). \end{aligned}$$

**Lemma 4** [14, 72] *For any invertible polynomial matrix  $\mathbf{X}(\mathbf{y})$  where  $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$ , the following equation is true .*

$$\frac{\partial \mathbf{X}(\mathbf{y})^{-1}}{\partial y_j} = -\mathbf{X}(\mathbf{y})^{-1} \frac{\partial \mathbf{X}(\mathbf{y})}{\partial y_j} \mathbf{X}(\mathbf{y})^{-1}, \quad \forall j.$$

## Chapter 3

# Control Design of IT2 T-S FMB Systems with Time-Varying Delay

In this chapter, the problems of stabilization for IT2 T-S fuzzy systems with time-varying delay and parameter uncertainties are investigated. The objective is to design an IT2 T-S fuzzy controller such that the closed-loop control system is asymptotically stable. The conditions for the existence of such a controller are delay dependent and membership function dependent in terms of LMIs. Based on a basic lemma, we formulate and solve the problem with more flexibility due to imperfect premise matching that the number of rules and premise membership functions are not necessary the same between the IT2 T-S fuzzy model and IT2 T-S fuzzy controller. A systematic approach making use of the information embedded in the lower and upper membership functions is employed to facilitate the stability analysis. A numerical example indicates the effectiveness of the derived results.

### 3.1 Problem Formulation and Preliminaries

The IT2 T-S FMB control system is shown in Fig. 3.1. An IT2 T-S fuzzy model is introduced to represent the nonlinear plant with time delay and parameter uncertainties. An IT2 T-S fuzzy controller is then introduced to control the nonlinear plant by closing the feedback loop.

#### 3.1.1 IT2 T-S Fuzzy Model with Time-Varying Delay

Consider a nonlinear system with time-varying delays and parameter uncertainties represented by the following IT2 T-S fuzzy model with lower and upper bound membership functions. **Plant Rule  $i$ :**

IF  $\theta_1(\mathbf{x}(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(\mathbf{x}(t))$  is  $\tilde{M}_{i2} \cdots$  and  $\theta_\Psi(\mathbf{x}(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t)) + \mathbf{B}_i \mathbf{u}(t) \\ \mathbf{x}(t) = \varphi(t), \quad t = [-\bar{d}, 0), \end{cases} \quad (3.1)$$

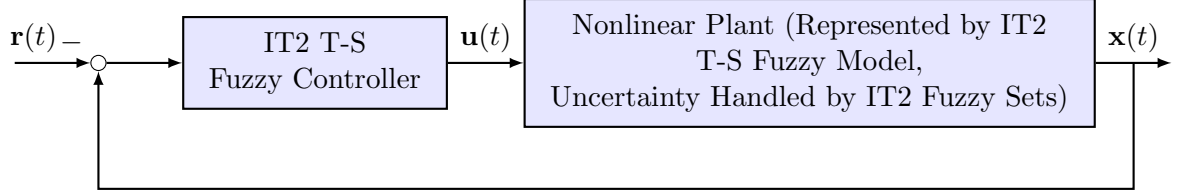


Figure 3.1: A block diagram of IT2 T-S FMB control system.

where  $\tilde{M}_{i\alpha}$  is an IT2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state,  $\mathbf{u}(t) \in \mathbb{R}^m$  is the control input,  $d(t)$  is the time-varying delay and satisfies  $d(t) \in (0, \bar{d}]$ ,  $\dot{d}(t) \leq m$ ,  $\bar{d}$  and  $m$  are known positive numbers. The initial condition associated with the delay is given in (3.1).  $\mathbf{A}_i$ ,  $\mathbf{B}_i$ ,  $\mathbf{A}_{di}$  are known matrices as system matrices, input matrices and delayed-state matrices, respectively. The inferred IT2 T-S fuzzy model is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t)) + \mathbf{B}_i \mathbf{u}(t)), \quad (3.2)$$

where (2.7) and (2.8) hold, in which  $\underline{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(\mathbf{x}(t)) + \bar{\alpha}_i(\mathbf{x}(t)) = 1$ .

### 3.1.2 IT2 T-S Fuzzy Controller

**Controller Rule  $j$ :**

IF  $\sigma_1(\mathbf{x}(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(\mathbf{x}(t))$  is  $\tilde{N}_{j2} \dots$  and  $\sigma_\Omega(\mathbf{x}(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$\mathbf{u}(t) = \mathbf{K}_j \mathbf{x}(t), \quad (3.3)$$

where  $\tilde{N}_{j\beta}$  is an IT2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ .  $\mathbf{K}_j$  are unknown feedback gains to be determined. The inferred IT2 T-S fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) \mathbf{K}_j \mathbf{x}(t), \quad (3.4)$$

where (2.14) and (2.15) hold, in which  $\underline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\beta}_j(\mathbf{x}(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(\mathbf{x}(t)) + \bar{\beta}_j(\mathbf{x}(t)) = 1$ .

### 3.1.3 IT2 T-S FMB Control System

With the plant and controller expression and the property of  $\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1$ ,  $\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1$ ,  $\sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) = 1$ , we can have the closed-loop control system as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t)) ((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))), \quad (3.5)$$

where  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t))$ . In addition,  $\tilde{h}_{ij}(\mathbf{x}(t))$  could be reconstructed as follows [35]:

$$\tilde{h}_{ij}(\mathbf{x}(t)) = \bar{\gamma}_{ij}(\mathbf{x}(t))\bar{h}_{ij}(\mathbf{x}(t)) + \underline{\gamma}_{ij}(\mathbf{x}(t))\underline{h}_{ij}(\mathbf{x}(t)), \quad (3.6)$$

in which  $\underline{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$  are functions with the property that  $\underline{\gamma}_{ij}(\mathbf{x}(t)) + \bar{\gamma}_{ij}(\mathbf{x}(t)) = 1$ , and  $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  are the piecewise linear membership function approximations of the upper and lower bound of  $\tilde{h}_{ij}(\mathbf{x}(t))$  with definitions below from [36]:

$$\bar{h}_{ij}(\mathbf{x}(t)) = \sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) \bar{\delta}_{ij i_1 i_2 \cdots i_n k}, \quad (3.7)$$

$$\underline{h}_{ij}(\mathbf{x}(t)) = \sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) \underline{\delta}_{ij i_1 i_2 \cdots i_n k}, \quad (3.8)$$

where  $0 \leq \underline{\delta}_{ij i_1 i_2 \cdots i_n k} \leq \bar{\delta}_{ij i_1 i_2 \cdots i_n k} \leq 1$  are scalars to be determined according to  $\tilde{h}_{ij}(\mathbf{x}(t))$ ,

$$0 \leq \underline{h}_{ij}(\mathbf{x}(t)) \leq \tilde{h}_{ij}(\mathbf{x}(t)) \leq \bar{h}_{ij}(\mathbf{x}(t)) \leq 1, \quad (3.9)$$

$v_{ri_r k}(x_r(t)) \in [0, 1]$ ,  $i_r = 1, 2$  and  $v_{r1k}(x_r(t)) + v_{r2k}(x_r(t)) = 1$ , otherwise  $v_{ri_r k}(x_r(t)) = 0$ ,  $\mathbf{x}(t) \in \Psi_k$ ,  $\cup_{k=1}^q \Psi_k = \Psi$  is the state space of interest.

**Remark 1** *With the above definitions, in the further stability analysis, we could use scalars  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$  to deal with the term  $\underline{h}_{ij}(\mathbf{x}(t))$  and  $\bar{h}_{ij}(\mathbf{x}(t))$  through  $\prod_{r=1}^n v_{ri_r k}(x_r(t))$  which are independent of  $i$  and  $j$ . In a word, the stability conditions involving the membership function information ( $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  as the upper and lower bound of  $\tilde{h}_{ij}(\mathbf{x}(t))$ ) could be achieved by scalars  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$ .*

## 3.2 Stability Analysis

For simplification reason, we denote  $\tilde{w}_i(\mathbf{x}(t))$ ,  $\tilde{m}_j(\mathbf{x}(t))$ ,  $\tilde{h}_{ij}(\mathbf{x}(t))$ ,  $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  as  $\tilde{w}_i$ ,  $\tilde{m}_j$ ,  $\tilde{h}_{ij}$ ,  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$ , respectively.

**Theorem 1** *Given constants  $m$ , positive scalar  $\bar{d}$ , predefined scalars  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  satisfying (3.7) and (3.8), if there exist positive matrices  $\mathbf{X}$ ,  $\bar{\mathbf{Y}}_{ij}$ ,  $\tilde{\mathbf{Q}}$ ,  $\tilde{\mathbf{Z}}$  and  $\tilde{\mathbf{T}}$  of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} -\tilde{\mathbf{Z}} & \tilde{\mathbf{T}} \\ * & -\tilde{\mathbf{Z}} \end{bmatrix} < 0, \quad (3.10)$$

$$\Xi_{ij} - \begin{bmatrix} \bar{\mathbf{Y}}_{ij} & 0 \\ 0 & 0 \end{bmatrix} < 0, \quad \forall i, j, \quad (3.11)$$

$$\sum_{i=1}^p \sum_{j=1}^c \left( \underline{\delta}_{ij i_1 i_2 \dots i_n k} \Xi_{ij} + (\bar{\delta}_{ij i_1 i_2 \dots i_n k} - \underline{\delta}_{ij i_1 i_2 \dots i_n k}) \right. \\ \left. \times \begin{bmatrix} \bar{\mathbf{Y}}_{ij} & 0 \\ 0 & 0 \end{bmatrix} \right) < 0, \quad (3.12) \\ \forall i, j, k, i_1, i_2, \dots, i_n,$$

where

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix}, \quad (3.13)$$

with

$$\begin{aligned} \Xi_{11} &= \mathbf{A}_i \mathbf{X} + \mathbf{B}_i \mathbf{N}_j + (\mathbf{A}_i \mathbf{X} + \mathbf{B}_i \mathbf{N}_j)^T \\ &\quad + \tilde{\mathbf{Q}} - \tilde{\mathbf{Z}}/\bar{d}, \end{aligned} \quad (3.14)$$

$$\Xi_{12} = \mathbf{A}_{di} \mathbf{X} + (\tilde{\mathbf{Z}} + \tilde{\mathbf{T}})/\bar{d}, \quad (3.15)$$

$$\Xi_{13} = -\tilde{\mathbf{T}}/\bar{d}, \quad (3.16)$$

$$\Xi_{14} = \sqrt{\bar{d}}(\mathbf{X} \mathbf{A}_i^T + \mathbf{Y}_j^T \mathbf{B}_i^T), \quad (3.17)$$

$$\Xi_{22} = (m-1)\tilde{\mathbf{Q}} - (\tilde{\mathbf{Z}} + \tilde{\mathbf{T}} + \tilde{\mathbf{T}}^T)/\bar{d}, \quad (3.18)$$

$$\Xi_{23} = (\tilde{\mathbf{Z}} + \tilde{\mathbf{T}})/\bar{d}, \quad (3.19)$$

$$\Xi_{24} = \sqrt{\bar{d}} \mathbf{X} \mathbf{A}_{di}^T, \quad (3.20)$$

$$\Xi_{33} = -\tilde{\mathbf{Z}}/\bar{d}, \quad (3.21)$$

$$\Xi_{34} = 0, \quad (3.22)$$

$$\Xi_{44} = \tilde{\mathbf{Z}} - 2\mathbf{X}, \quad (3.23)$$

then the closed-loop control system (3.5) is asymptotically stable. Moreover, the IT2 T-S fuzzy controller gains can be obtained by  $\mathbf{K}_j = \mathbf{N}_j \mathbf{X}^{-1}$ .

We need to revisit a fundamental lemma to be used in the following proof.

**Lemma 5** [77] For matrix  $\mathbf{N} = \begin{bmatrix} -\mathbf{R} & \mathbf{L} \\ * & -\mathbf{R} \end{bmatrix} \leq 0$ ,  $d(t) \in (0, \bar{d}]$ , and a vector function  $\dot{\mathbf{x}} : [-\bar{d}, 0) \rightarrow \mathbb{R}^n$  such that the integration in the following inequality is well defined, then it holds that

$$-\bar{d} \int_{t-\bar{d}}^t \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds \leq v^T(t) \mathbf{W} v(t), \quad (3.24)$$

where

$$\mathbf{W} = \begin{bmatrix} -\mathbf{R} & \mathbf{R} + \mathbf{L} & -\mathbf{L} \\ * & -2\mathbf{R} + \mathbf{L} + \mathbf{L}^T & \mathbf{R} + \mathbf{L} \\ * & * & -\mathbf{R} \end{bmatrix}, \quad (3.25)$$

$$v^T(t) = [\mathbf{x}^T(t) \ \mathbf{x}^T(t - d(t)) \ \mathbf{x}^T(t - \bar{d})]. \quad (3.26)$$

**Proof** Consider a candidate of Lyapunov-Krasovskii functional with symmetric positive definite matrices  $\mathbf{P}$ ,  $\mathbf{Q}$  and  $\mathbf{Z}$  as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (3.27)$$

$$V_1(t) = \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t), \quad (3.28)$$

$$V_2(t) = \int_{t-d(t)}^t \mathbf{x}^T(s) \mathbf{Q} \mathbf{x}(s) ds, \quad (3.29)$$

$$V_3(t) = \int_{-\bar{d}}^0 \int_{t+\theta}^t \dot{\mathbf{x}}^T(s) \mathbf{Z} \dot{\mathbf{x}}(s) ds d\theta. \quad (3.30)$$

Along the trajectories of the closed-loop control system, the corresponding time derivative of  $V(t)$  is given by

$$\begin{aligned} \dot{V}_1(t) &= 2\mathbf{x}^T(t) \mathbf{P} \dot{\mathbf{x}}(t) \\ &= 2 \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \\ &\quad \times \mathbf{P}((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))), \\ \dot{V}_2(t) &= \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) - (1 - \dot{d}(t)) \mathbf{x}^T(t - d(t)) \mathbf{Q} \mathbf{x}(t - d(t)) \\ &\leq \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) - (1 - m) \mathbf{x}^T(t - d(t)) \mathbf{Q} \mathbf{x}(t - d(t)), \\ \dot{V}_3(t) &= \bar{d} \dot{\mathbf{x}}^T(t) \mathbf{Z} \dot{\mathbf{x}}(t) - \int_{t-\bar{d}}^t \dot{\mathbf{x}}^T(s) \mathbf{Z} \dot{\mathbf{x}}(s) ds. \end{aligned}$$

By applying Lemma 5,  $\dot{V}_3(t)$  can be expressed as

$$\begin{aligned} \dot{V}_3(t) &\leq \bar{d} \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \\ &\quad \times ((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t)))^T \mathbf{Z} \\ &\quad \times ((\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))) \\ &\quad + \frac{1}{\bar{d}} v^T(t) \begin{bmatrix} -\mathbf{Z} & \mathbf{Z} + \mathbf{T} & -\mathbf{T} \\ * & -2\mathbf{Z} - \mathbf{T} - \mathbf{T}^T & \mathbf{Z} + \mathbf{T} \\ * & * & -\mathbf{Z} \end{bmatrix} v(t), \end{aligned}$$

with  $v^T(t) = [\mathbf{x}^T(t) \ \mathbf{x}^T(t - d(t)) \ \mathbf{x}^T(t - \bar{d})]$  and subject to  $\begin{bmatrix} -\mathbf{Z} & \mathbf{T} \\ * & -\mathbf{Z} \end{bmatrix} \leq 0$ .

Rewrite  $\dot{V}(t)$  with  $\mathbf{\Omega}_{ij} = \begin{bmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} & -\mathbf{T}/\bar{d} \\ * & \mathbf{\Lambda}_{22} & (\mathbf{Z} + \mathbf{T})/\bar{d} \\ * & * & -\mathbf{Z}/\bar{d} \end{bmatrix}$ , where

$$\begin{aligned} \mathbf{\Lambda}_{11} &= \mathbf{P}(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j) + (\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j)^T \mathbf{P} + \mathbf{Q} - \mathbf{Z}/\bar{d} \\ &\quad + \bar{d}(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j)^T \mathbf{Z}(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j), \\ \mathbf{\Lambda}_{12} &= (\mathbf{Z} + \mathbf{T})/\bar{d} + \mathbf{P} \mathbf{A}_{di} + \bar{d}(\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j)^T \mathbf{Z} \mathbf{A}_{di}, \\ \mathbf{\Lambda}_{22} &= (m - 1)\mathbf{Q} - (2\mathbf{Z} + \mathbf{T} + \mathbf{T}^T)/\bar{d} + \bar{d} \mathbf{A}_{di}^T \mathbf{Z} \mathbf{A}_{di}. \end{aligned}$$

Then we get

$$\begin{aligned} \dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} v^T(t) \mathbf{\Omega}_{ij} v(t) \\ &= \sum_{i=1}^p \sum_{j=1}^c (\underline{h}_{ij} + (\tilde{h}_{ij} - \underline{h}_{ij})) v^T(t) \mathbf{\Omega}_{ij} v(t) \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \underline{h}_{ij} v^T(t) \mathbf{\Omega}_{ij} v(t) + (\bar{h}_{ij} - \underline{h}_{ij}) v^T(t) \mathbf{Y}_{ij} v(t) \\ &= \sum_{i=1}^p \sum_{j=1}^c v^T(t) (\underline{h}_{ij} \mathbf{\Omega}_{ij} + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{Y}_{ij}) v(t), \end{aligned}$$

with

$$\begin{aligned} \mathbf{Y}_{ij} &\geq 0, \\ \mathbf{Y}_{ij} &\geq \mathbf{\Omega}_{ij}, \end{aligned}$$

The stability condition for the closed-loop control system would be

$$\sum_{i=1}^p \sum_{j=1}^c v^T(t) (\underline{h}_{ij} \mathbf{\Omega}_{ij} + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{Y}_{ij}) v(t) < 0. \quad (3.31)$$

Recalling that  $\sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) = 1$ , with (3.7) and (3.8), by using Schur Complement and congruence transformation, we can get the conditions as stated in the theorem with  $\mathbf{X} = \mathbf{P}^{-1}$ ,  $\mathbf{K}_j \mathbf{X} = \mathbf{N}_j$ ,  $\tilde{\mathbf{Q}} = \mathbf{X} \mathbf{Q} \mathbf{X}$ ,  $\tilde{\mathbf{Z}} = \mathbf{X} \mathbf{Z} \mathbf{X}$ ,  $\tilde{\mathbf{T}} = \mathbf{X} \mathbf{T} \mathbf{X}$ ,  $\bar{\mathbf{Y}}_{ij} = \text{diag}\{\mathbf{X}, \mathbf{X}, \mathbf{X}\} \mathbf{Y}_{ij} \text{diag}\{\mathbf{X}, \mathbf{X}, \mathbf{X}\}$ .

**Remark 2** Theorem 1 introduces membership functions  $\tilde{h}_{ij}$  which are reconstructed by the upper bound  $\bar{h}_{ij}$  and lower bound  $\underline{h}_{ij}$ . Moreover  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$  could be expressed by predefined scalars  $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  in the form of (3.7) and (3.8). This



will allow us to just check conditions at certain points ( $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$ ) rather than every point of the upper bound  $\bar{h}_{ij}$  and lower bound  $\underline{h}_{ij}$ .

**Remark 3** As Theorem 1 involves the information of the membership functions in control design, it is an MFD method which is less conservative than the MFI method. While Theorem 1 could be reduced to the following theorem which is MFI for control design.

**Theorem 2** Given constants  $m$ , positive scalar  $\bar{d}$ , if there exist positive matrices  $\mathbf{X}$ ,  $\tilde{\mathbf{Q}}$ ,  $\tilde{\mathbf{Z}}$  and  $\tilde{\mathbf{T}}$  of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} -\tilde{\mathbf{Z}} & \tilde{\mathbf{T}} \\ * & -\tilde{\mathbf{Z}} \end{bmatrix} < 0, \quad (3.32)$$

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} < 0, \quad \forall i, j, \quad (3.33)$$

the elements in  $\Xi_{ij}$  are the same as stated in Theorem 1. Then the closed-loop control system (3.5) is asymptotically stable. Moreover, the IT2 T-S fuzzy controller gains can be obtained by  $\mathbf{K}_j = \mathbf{N}_j \mathbf{X}^{-1}$ .

**Remark 4** In the derivation of Theorem 1, we introduce slack matrices  $\mathbf{Y}_{ij}$  to bring more flexibility. We can include even more slack matrices based on some inequalities and equalities, but this will lead to high computational demand.

**Remark 5** It could be noted that dividing the region of  $x$  into more partitions could further reduce the conservatism. The more upper and lower bounds of the membership functions involved in could lead to more relaxed results while the computation burden would be heavier.

**Remark 6** Theorems 1 and 2 could be modified to tackle control systems without time-varying delay by removing delay related terms in  $V(t)$  following the similar derivation.

### 3.3 Simulation Example

In this section, a numerical example will be presented to demonstrate the potential and validity of our developed theoretical results.

Consider a three-rule IT2 T-S fuzzy model in the form of (3.2) with  $\mathbf{A}_1 = \begin{bmatrix} 2.78 & -5.63 \\ 0.01 & 0.33 \end{bmatrix}$ ,  $\mathbf{A}_2 = \begin{bmatrix} 0.2 & -3.22 \\ 0.35 & 0.12 \end{bmatrix}$ ,  $\mathbf{A}_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}$ ,  $\mathbf{B}_1 = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$ ,

$$\mathbf{B}_2 = \begin{bmatrix} 8 & 0 \end{bmatrix}^T, \mathbf{B}_3 = \begin{bmatrix} -b+6 & -1 \end{bmatrix}^T, \mathbf{A}_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \mathbf{A}_{d2} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix},$$

$$\mathbf{A}_{d3} = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.6 \end{bmatrix}. \quad a \text{ and } b \text{ are constant parameters, } \mathbf{x} = [x_1 \ x_2]^T. \quad d(t) = 0.1\bar{d}(1 + 9\sin^2 t), \quad m = 0.9\bar{d}. \quad \varphi(t) = 0 \text{ when } t \in [-\bar{d}, 0).$$

The membership functions for the plant (3.2) are chosen as  $\tilde{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+\eta(t))})$ ,  $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ ,  $\tilde{w}_3(x_1) = 1/(1 + e^{-(x_1-4+\eta(t))})$ . Due to parameter uncertainty  $\eta(t)$ , the membership functions are uncertain grades of membership. The lower and upper membership functions are chosen as  $\underline{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+0.25)})$ ,  $\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1) - \underline{w}_3(x_1)$ ,  $\underline{w}_3(x_1) = 1/(1 + e^{-(x_1-4-0.25)})$ ,  $\bar{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4-0.25)})$ ,  $\bar{w}_3(x_1) = 1/(1 + e^{-(x_1-4+0.25)})$ ,  $\underline{w}_2(x_1) = 1 - \bar{w}_1(x_1) - \bar{w}_3(x_1)$ .

The lower and upper membership functions for the controller (3.4) are chosen as  $\underline{m}_1(x_1) = 1 - 1/e^{-(x_1+0.15)/2}$ ,  $\bar{m}_1(x_1) = 1 - 1/e^{-(x_1-0.15)/2}$ ,  $\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$ ,  $\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$ . From (2.14), we can get  $\tilde{m}_j(x_1)$ . Set  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ , we can get stability regions by conditions in Theorem 1 subject to different values of  $a$  and  $b$ . We consider the grades of membership are capped and focus on the region  $x_1 \in [-10, 10]$ . We consider  $132 \leq a \leq 140$  at the interval of one and  $5 \leq b \leq 60$  at the interval of five. With  $\bar{d} = 0.19$ , we can see the stability region given by Theorem 1 (MFD method) indicated by “o” is larger than that given by Theorem 2 (MFI method) indicated by “+”. The result is shown in Fig. 3.1. When the upper bound of the delay increases, for example  $\bar{d} = 1$ , MFI method would not give any feasible solution, and the stability region given by Theorem 1 (MFD method) indicated by “o” is shown in Fig. 3.2 as  $132 \leq a \leq 140$  at the interval of one and  $5 \leq b \leq 60$  at the interval of five. These reveal the less of conservatism of the proposed MFD method given in the chapter.

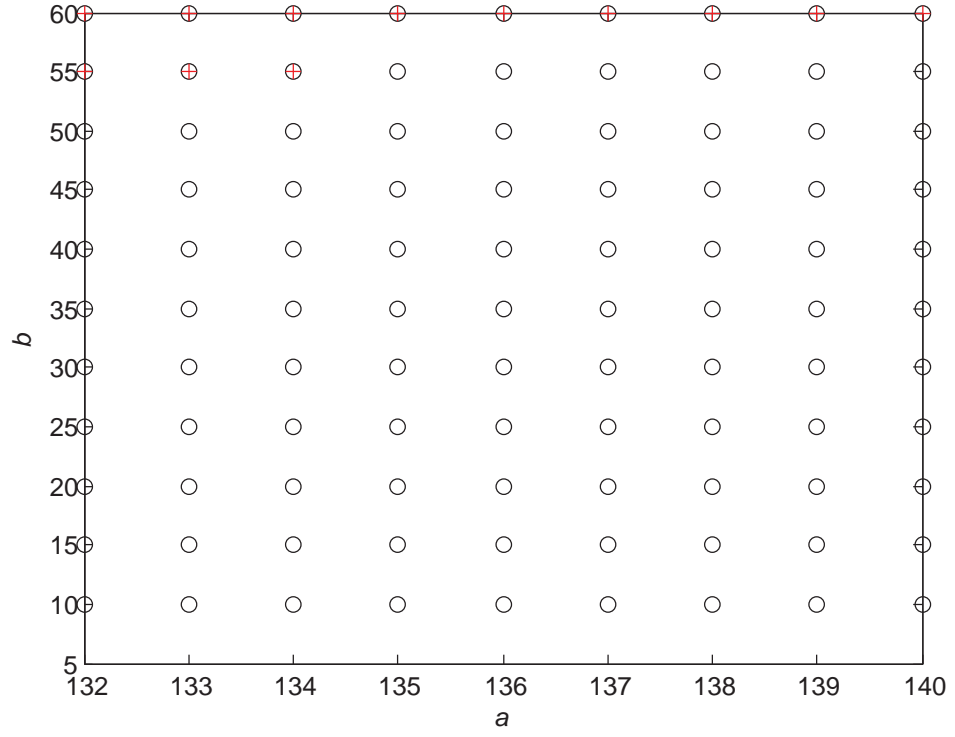


Figure 3.2: Stability regions given by conditions in Theorem 1 ( $\circ$ , 99 points) and Theorem 2 ( $+$ , 12 points) with  $\bar{d} = 0.19$ .

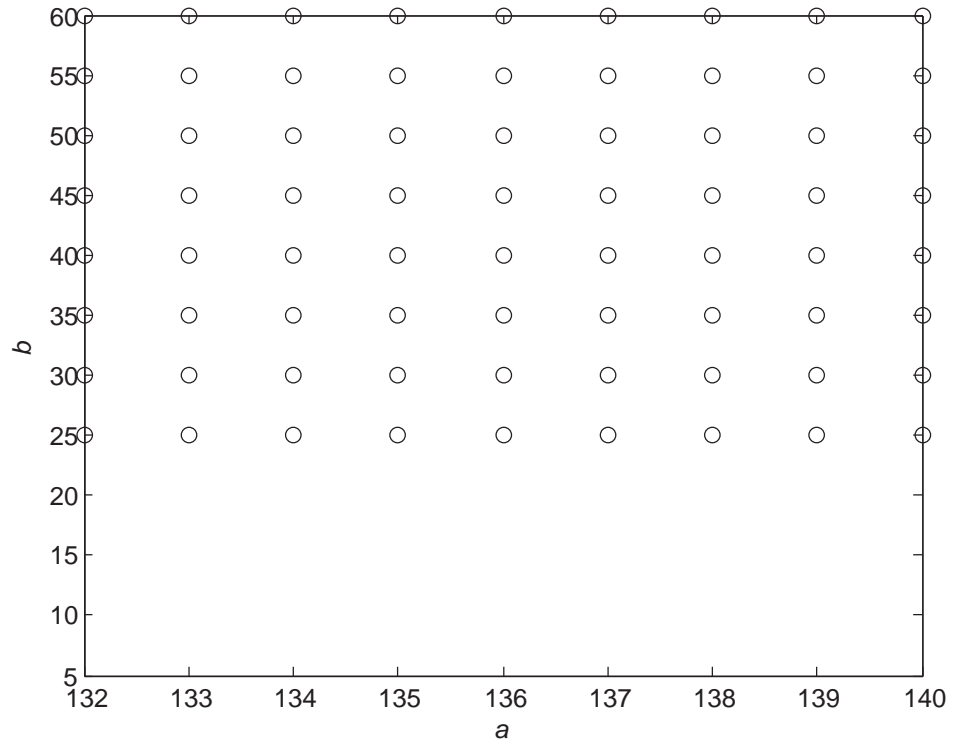


Figure 3.3: Stability regions given by conditions in Theorem 1 ( $\circ$ , 72 points) and Theorem 2 ( $+$ , 0 point) with  $\bar{d} = 1$ . In this case Theorem 2 is not able to find stability region

With  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $\mathbf{x}(0) = [-5 \ 0]^T$ ,  $\bar{d} = 0.19$ , Fig.

3.3 gives the state response of the closed-loop control system which is asymptotically stable with the controller gains  $\mathbf{K}_1 = \begin{bmatrix} -4.823987 & -0.042365 \end{bmatrix}$ ,  $\mathbf{K}_2 = \begin{bmatrix} -5.283737 & -0.271163 \end{bmatrix}$  and  $\mathbf{P} = \begin{bmatrix} 0.8758 & -0.4786 \\ -0.4786 & 3.5184 \end{bmatrix}$ . Fig. 3.4 gives the state response of the closed-loop control system with the same parameters used in Fig. 3.3 except for  $\bar{d} = 1$  with the controller gains  $\mathbf{K}_1 = \begin{bmatrix} -0.577141 & 0.690671 \end{bmatrix}$ ,  $\mathbf{K}_2 = \begin{bmatrix} -5.396162 & -0.340777 \end{bmatrix}$  and  $\mathbf{P} = \begin{bmatrix} 0.0851 & -0.1199 \\ -0.1199 & 0.7701 \end{bmatrix}$ .

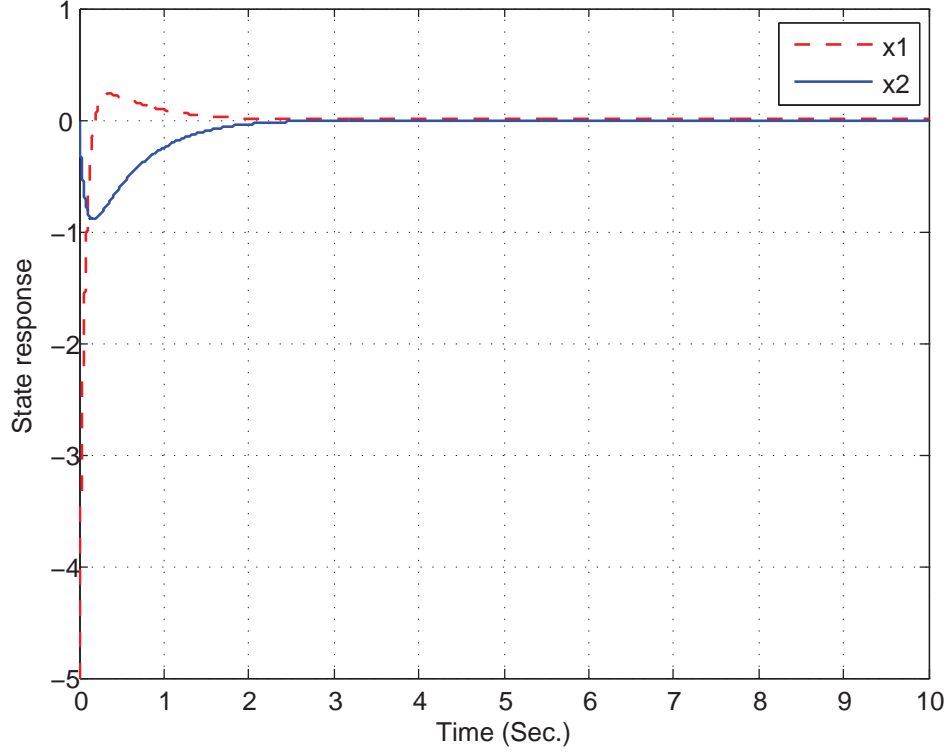


Figure 3.4: State response of the closed-loop control system with  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $\mathbf{x}(0) = [-5 \ 0]^T$ ,  $\bar{d} = 0.19$ .

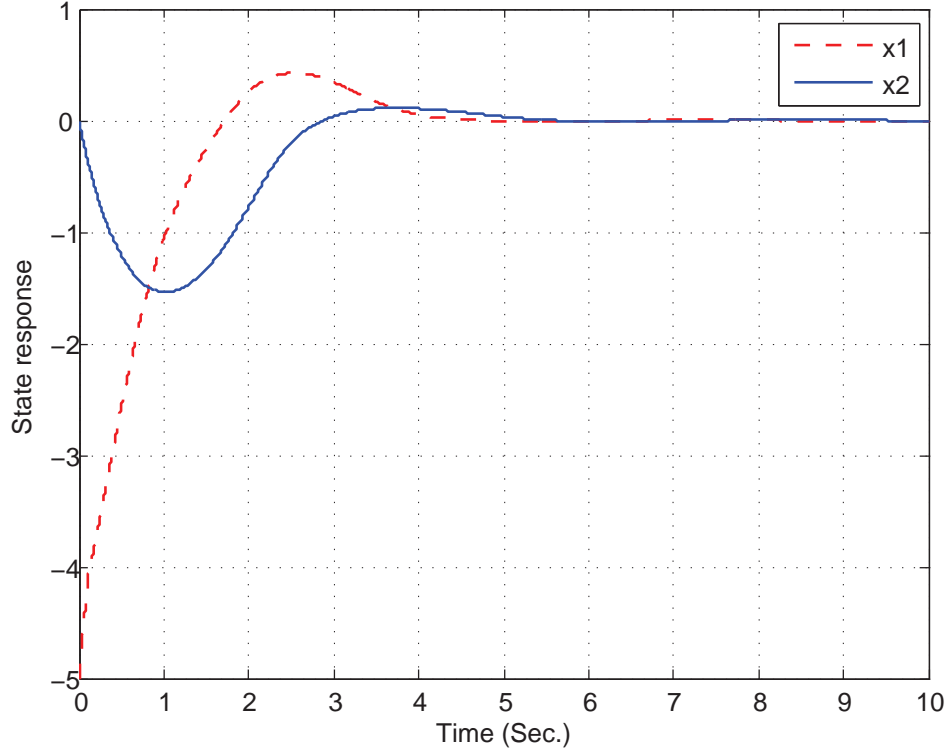


Figure 3.5: State response of the closed-loop control system with  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $\mathbf{x}(0) = [-5 \ 0]^T$ ,  $\bar{d} = 1$ .

**Remark 7** From Figure 3.4 and Figure 3.5, we can see how the time delay affect the result. The controller dealing the system of a short delay has a fast response and the controller dealing the system of a long delay has a slow and sluggish response.

### 3.4 Conclusion

The stability of IT2 T-S FMB control systems with time-varying delay and parameter uncertainties is investigated in this chapter. We have proposed an IT2 T-S fuzzy state feedback controller to ensure the asymptotic stability of the closed-loop control system under imperfect premise matching. This MFD method shares more design flexibility, because it is not required that the IT2 T-S fuzzy controller and IT2 T-S fuzzy plant have the same premise membership function and/or number of fuzzy rules. The stability conditions come in LMIs form and include the information of the membership functions to be more relaxed than MFI method. A numerical example is presented to show the effectiveness of the proposed approach.

# Chapter 4

## Control Design of IT2 T-S FMB Systems with Actuator Saturation

In this Chapter, the problems of asymptotical stabilization for IT2 T-S fuzzy systems with actuator saturation are investigated. The sufficient conditions for the existence of the IT2 T-S fuzzy controller are in terms of SOS. The problem is formulated and solved with more flexibility due to imperfect premise matching that the number of rules and premise membership functions of the fuzzy controller are not necessarily the same as the ones of the fuzzy model. The actuator saturation is depicted and dealt with contractively invariant ellipsoid. Piecewise linear membership functions enclosing the original lower and upper membership functions are employed to facilitate the stability analysis. A numerical example indicates the effectiveness of the derived results.

### 4.1 Problem Formulation and Preliminaries

In this section we consider a nonlinear system with actuator saturation and parameter uncertainties represented by the IT2 T-S fuzzy model with lower and upper bound membership functions like Chapter 2. We just focus on the actuator saturation problem in this work, so the underlying system we study is delay-free. From now on, unless specified, the settings in this chapter would be the same as the ones in Chapter 2. The IT2 T-S FMB control system with actuator saturation is shown in Fig. 4.1. An IT2 T-S fuzzy model is introduced to represent the nonlinear plant with parameter uncertainties. An IT2 T-S fuzzy controller is then introduced to control the nonlinear plant by closing the feedback loop, where the output of the controller is saturated before input to the plant.

#### 4.1.1 IT2 T-S Fuzzy Model with Actuator Saturation

**Plant Rule  $i$ :**

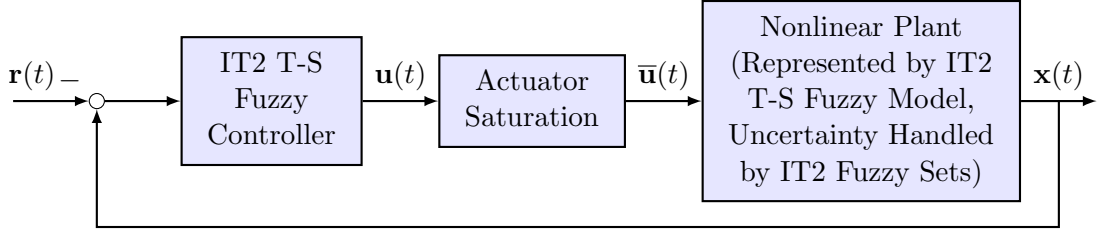


Figure 4.1: A block diagram of IT2 T-S FMB control system with actuator saturation.

IF  $\theta_1(\mathbf{x}(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(\mathbf{x}(t))$  is  $\tilde{M}_{i2} \cdots$  and  $\theta_\Psi(\mathbf{x}(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \bar{\mathbf{u}}(t), \quad (4.1)$$

where  $\tilde{M}_{i\alpha}$  is an IT2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state,  $\bar{\mathbf{u}}(t) = [\bar{u}_1(t), \dots, \bar{u}_m(t)]^T = [\text{sat}(u_1(t)), \dots, \text{sat}(u_m(t))]^T = \text{sat}(\mathbf{u}(t)) \in \mathbb{R}^m$  is the saturated control input. The inferred IT2 T-S fuzzy model is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \bar{\mathbf{u}}(t)), \quad (4.2)$$

where (2.7) and (2.8) hold, in which  $\underline{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(\mathbf{x}(t)) + \bar{\alpha}_i(\mathbf{x}(t)) = 1$ .

*Definition 1:* The actuator saturation could be defined as follows:

$$\bar{u}_l(t) = \text{sat}(u_l) = \begin{cases} u_{lH} & u_{lH} < u_l \\ u_l & u_{lL} \leq u_l \leq u_{lH}, \\ u_{lL} & u_l < u_{lL} \end{cases} \quad (4.3)$$

where  $l = 1, 2, \dots, m$  and  $u_{lH} > 0 > u_{lL}$ .  $u_{lH}$  and  $u_{lL}$  are known scalars.

We introduce a parameter  $\epsilon$ , which is  $0 < \epsilon < 1$ , to make sure the saturation map  $\text{sat}(\cdot)$  is inside the sector  $(\epsilon, 1)$ . Then we have

$$\frac{u_{lL}}{\epsilon} \leq u_l \leq \frac{u_{lH}}{\epsilon}. \quad (4.4)$$

For simplicity, if we set  $u_{lH} = -u_{lL}$ , then we have

$$|u_l| \leq \frac{u_{lH}}{\epsilon}. \quad (4.5)$$

With all the settings and definition above, one can get

$$\|\bar{\mathbf{u}}(t) - \frac{1+\epsilon}{2} \mathbf{u}(t)\| \leq \frac{1-\epsilon}{2} \|\mathbf{u}(t)\|. \quad (4.6)$$

#### 4.1.2 IT2 T-S Fuzzy Controller

Controller Rule  $j$ :

IF  $\sigma_1(\mathbf{x}(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(\mathbf{x}(t))$  is  $\tilde{N}_{j2} \cdots$  and  $\sigma_\Omega(\mathbf{x}(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$\mathbf{u}(t) = \mathbf{K}_j \mathbf{x}(t), \quad (4.7)$$

where  $\tilde{N}_{j\beta}$  is an IT2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ .  $\mathbf{K}_j$  are unknown feedback gains to be determined. The inferred IT2 T-S fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) \mathbf{K}_j \mathbf{x}(t), \quad (4.8)$$

where (2.14) and (2.15) hold, in which  $\underline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\beta}_j(\mathbf{x}(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(\mathbf{x}(t)) + \bar{\beta}_j(\mathbf{x}(t)) = 1$ .

### 4.1.3 IT2 T-S FMB Control System

With the plant and controller expressions, we can have the closed-loop control system as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t)) \left( (\mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{B}_i \mathbf{R} \right), \quad (4.9)$$

where  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t))$ ,  $\mathbf{R} = \bar{\mathbf{u}}(t) - \frac{1+\epsilon}{2} \mathbf{u}(t)$ . In addition,  $\tilde{h}_{ij}(\mathbf{x}(t))$  could be reconstructed as (3.6) in which  $\underline{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$  are functions with the property that  $\underline{\gamma}_{ij}(\mathbf{x}(t)) + \bar{\gamma}_{ij}(\mathbf{x}(t)) = 1$ , and  $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  are the piecewise linear membership function approximations of the upper and lower bound of  $\tilde{h}_{ij}(\mathbf{x}(t))$  with definitions in (3.7) and (3.8) where  $0 \leq \underline{\delta}_{ij i_1 i_2 \dots i_n k} \leq \bar{\delta}_{ij i_1 i_2 \dots i_n k} \leq 1$  are scalars to be determined according to  $\tilde{h}_{ij}(\mathbf{x}(t))$ , where (3.9) holds,  $v_{ri_r k}(x_r(t)) \in [0, 1]$ ,  $i_r = 1, 2$  and  $v_{r1k}(x_r(t)) + v_{r2k}(x_r(t)) = 1$ , otherwise  $v_{ri_r k}(x_r(t)) = 0$ ,  $\mathbf{x}(t) \in \Psi_k$ ,  $\cup_{k=1}^q \Psi_k = \Psi$  is the state space of interest.

*Definition 2:* Let an ellipsoid  $\mathbf{\Lambda}_1$  and a positive scalar function  $V(\mathbf{x}(t))$  be defined as follows, respectively,

$$\mathbf{\Lambda}_1 = \{ \mathbf{x}(t) | \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) \leq 1 \}, \quad (4.10)$$

and

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t), \quad (4.11)$$

where  $\mathbf{P} \in \mathbb{R}^{n \times n}$  denotes a positive definite matrix. An ellipsoid  $\mathbf{\Lambda}_1$ , which is inside the domain of attraction, is said to be contractively invariant [69] if

$$\dot{V}(\mathbf{x}(t)) < 0, \forall \mathbf{x}(t) \in \mathbf{\Lambda}_1 \setminus \{0\}.$$



The saturation constraint on the input  $u_l$  could be expressed as follows

$$|\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) \mathbf{K}_j^{(l)} \mathbf{x}(t)| \leq \frac{u_{lH}}{\epsilon}, \quad (4.12)$$

where  $\mathbf{K}_j^{(l)}$  denotes the  $l$ th row of  $\mathbf{K}_j$ . Let

$$\mathbf{\Lambda}_2 = \left\{ \mathbf{x}(t) | \mathbf{x}^T(t) (\mathbf{K}_j^{(l)})^T (\mathbf{K}_j^{(l)}) \mathbf{x}(t) \leq \left(\frac{u_{lH}}{\epsilon}\right)^2 \right\}. \quad (4.13)$$

It is required that  $\mathbf{x}(t) \in \mathbf{\Lambda}_1 \subset \mathbf{\Lambda}_2$ . The equivalent condition for  $\mathbf{x}(t) \in \mathbf{\Lambda}_1 \subset \mathbf{\Lambda}_2$  is

$$(\mathbf{K}_j^{(l)}) \mathbf{P}^{-1} (\mathbf{K}_j^{(l)})^T \leq \left(\frac{u_{lH}}{\epsilon}\right)^2. \quad (4.14)$$

## 4.2 Stability Analysis

For simplification reason, we denote  $\tilde{w}_i(\mathbf{x}(t))$ ,  $\tilde{m}_j(\mathbf{x}(t))$ ,  $\tilde{h}_{ij}(\mathbf{x}(t))$ ,  $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  as  $\tilde{w}_i$ ,  $\tilde{m}_j$ ,  $\tilde{h}_{ij}$ ,  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$ , respectively.

We need to revisit two lemmas to be used in the following proof for the sufficient stability conditions.

**Lemma 6** [78] *For any two matrices  $\mathbf{X}$  and  $\mathbf{Y}$ , one has*

$$\mathbf{X}^T \mathbf{Y} + \mathbf{Y}^T \mathbf{X} \leq \mathbf{X}^T \mathbf{X} + \mathbf{Y}^T \mathbf{Y}.$$

**Lemma 7** [79] *Suppose that matrices  $\mathbf{M}_i \in \mathbb{R}^{m \times n}$ ,  $i = 1, 2, \dots, r$ , and a positive matrix  $\mathbf{Q} \in \mathbb{R}^{m \times m}$  are given. If  $\sum_{i=1}^r p_i = 1$  and  $0 \leq p_i \leq 1$ , then*

$$\left( \sum_{i=1}^r p_i \mathbf{M}_i \right)^T \mathbf{Q} \left( \sum_{i=1}^r p_i \mathbf{M}_i \right) \leq \sum_{i=1}^r p_i \mathbf{M}_i^T \mathbf{Q} \mathbf{M}_i.$$

**Theorem 3** *Given predefined scalars  $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  satisfying (3.7) and (3.8), if there exist matrices  $\mathbf{X} = \mathbf{X}^T$ ,  $\mathbf{Y}_{ij} = \mathbf{Y}_{ij}^T$ ,  $\mathbf{\Omega}_{ij}$  and  $\mathbf{N}_j$  of appropriate dimensions such that the following SOS-based conditions hold:*

$$\begin{aligned} & v_1^T (\mathbf{X} - \varepsilon_1 \mathbf{I}) v_1 && \text{is SOS;} \\ & v_2^T (\mathbf{Y}_{ij} - \varepsilon_2 \mathbf{I}) v_2 && \text{is SOS;} \\ & v_2^T (\mathbf{Y}_{ij} - \mathbf{\Omega}_{ij} - \varepsilon_3 \mathbf{I}) v_2 && \text{is SOS;} \\ & -v_2^T \left( \sum_{i=1}^p \sum_{j=1}^c (\delta_{ij i_1 i_2 \dots i_n k} \mathbf{\Omega}_{ij} \right. && \\ & \left. + (\bar{\delta}_{ij i_1 i_2 \dots i_n k} - \underline{\delta}_{ij i_1 i_2 \dots i_n k}) \mathbf{Y}_{ij} + \varepsilon_4 \mathbf{I}) v_2 \right. && \text{is SOS;} \\ & \left. v_2^T \left( \begin{bmatrix} \left(\frac{1+\epsilon}{2} \cdot \frac{u_{lH}}{\epsilon}\right)^2 & \frac{1+\epsilon}{2} \mathbf{N}_j^{(l)} \\ * & \mathbf{X} \end{bmatrix} - \varepsilon_5 \mathbf{I} \right) v_2 \right. && \text{is SOS;} \end{aligned} \quad (4.15)$$

where

$$\Omega_{ij} = \begin{bmatrix} \mathbf{W}_{ij} + \mathbf{B}_i \mathbf{B}_i^T & \frac{1-\epsilon}{2} \mathbf{N}_j^T \\ * & -\mathbf{I} \end{bmatrix}, \quad (4.16)$$

$\mathbf{W}_{ij} = \mathbf{A}_i \mathbf{X} + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{N}_j + (\mathbf{A}_i \mathbf{X} + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{N}_j)^T$ ,  $i = 1, 2, \dots, p$ ,  $j = 1, 2, \dots, c$ ,  $l = 1, 2, \dots, m$ .  $\mathbf{N}_j^{(l)}$  denotes the  $l$ th row of  $\mathbf{N}_j$ .  $v_1, v_2$  are arbitrary vectors independent of  $x$ .  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$  and  $\varepsilon_5$  are predefined positive scalars. Then the closed-loop control system (4.9) is asymptotically stable. In addition, the IT2 T-S fuzzy controller gains can be obtained by  $\mathbf{K}_j = \mathbf{N}_j \mathbf{X}^{-1}$ .

**Proof** Consider a candidate of Lyapunov functional as the one described in Definition 2

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t).$$

Along the trajectories of the closed-loop control system, the corresponding time derivative of  $V(\mathbf{x}(t))$  is given by

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &= \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \left[ \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right)^T \mathbf{P} \right. \\ &\quad \left. + \mathbf{P} \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right) \right] \mathbf{x}(t) \\ &\quad + \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j [\mathbf{R}^T \mathbf{B}_i^T \mathbf{P} \mathbf{x}(t) + \mathbf{x}^T(t) \mathbf{P} \mathbf{B}_i \mathbf{R}] \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \left[ \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right)^T \mathbf{P} \right. \\ &\quad \left. + \mathbf{P} \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right) \right] \mathbf{x}(t) \\ &\quad + \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j [\mathbf{x}^T(t) \mathbf{P} \mathbf{B}_i \mathbf{B}_i^T \mathbf{P} \mathbf{x}(t) + \mathbf{R}^T \mathbf{R}] \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \left[ \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right)^T \mathbf{P} \right. \\ &\quad \left. + \mathbf{P} \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right) \right] \mathbf{x}(t) \\ &\quad + \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \mathbf{P} \mathbf{B}_i \mathbf{B}_i^T \mathbf{P} \mathbf{x}(t) \\ &\quad + \left( \frac{1-\epsilon}{2} \right)^2 \left( \sum_{j=1}^c \tilde{m}_j \mathbf{K}_j \mathbf{x}(t) \right)^2 \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \left[ \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right)^T \mathbf{P} \right. \\ &\quad \left. + \mathbf{P} \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right) \right] \mathbf{x}(t) \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \mathbf{P} \mathbf{B}_i \mathbf{B}_i^T \mathbf{P} \mathbf{x}(t) \\
& + \left(\frac{1-\epsilon}{2}\right)^2 \sum_{j=1}^c \tilde{m}_j \mathbf{x}^T(t) \mathbf{K}_j^T \mathbf{K}_j \mathbf{x}(t) \\
& = \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \left[ \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right)^T \mathbf{P} \right. \\
& \quad \left. + \mathbf{P} \left( \mathbf{A}_i + \frac{1+\epsilon}{2} \mathbf{B}_i \mathbf{K}_j \right) \right. \\
& \quad \left. + \mathbf{P} \mathbf{B}_i \mathbf{B}_i^T \mathbf{P} + \left(\frac{1-\epsilon}{2}\right)^2 \mathbf{K}_j^T \mathbf{K}_j \right] \mathbf{x}(t).
\end{aligned}$$

By using Schur Complement and setting  $\mathbf{X}^{-1} = \mathbf{P}$ , we can rewrite the stability condition as

$$M(\mathbf{x}(t)) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} \mathbf{x}^T(t) \boldsymbol{\Omega}_{ij} \mathbf{x}(t) < 0. \quad (4.17)$$

In order to include the information of the membership functions, we introduce slack matrices  $\mathbf{Y}_{ij}$  into the analysis with the property  $\mathbf{Y}_{ij} > 0$ ,  $\mathbf{Y}_{ij} > \boldsymbol{\Omega}_{ij}$ . Then we have

$$\begin{aligned}
M(\mathbf{x}(t)) &= \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} \mathbf{x}^T(t) \boldsymbol{\Omega}_{ij} \mathbf{x}(t) \\
&= \sum_{i=1}^p \sum_{j=1}^c (\underline{h}_{ij} + (\tilde{h}_{ij} - \underline{h}_{ij})) \mathbf{x}^T(t) \boldsymbol{\Omega}_{ij} \mathbf{x}(t) \\
&\leq \sum_{i=1}^p \sum_{j=1}^c \underline{h}_{ij} \mathbf{x}^T(t) \boldsymbol{\Omega}_{ij} \mathbf{x}(t) \\
&\quad + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{x}^T(t) \mathbf{Y}_{ij} \mathbf{x}(t) \\
&= \sum_{i=1}^p \sum_{j=1}^c \mathbf{x}^T(t) (\underline{h}_{ij} \boldsymbol{\Omega}_{ij} + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{Y}_{ij}) \mathbf{x}(t).
\end{aligned}$$

Recalling  $\sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) = 1$ , with (3.7) and (3.8) and the constraint on the input, we can convert the above conditions into the SOS-based ones stated in the theorem. The proof is complete.

**Remark 8** *Theorem 3 introduces membership functions  $\tilde{h}_{ij}$  which are reconstructed by the upper bound  $\bar{h}_{ij}$  and lower bound  $\underline{h}_{ij}$ . Moreover  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$  could be expressed by predefined scalars  $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  in the form of (3.7) and (3.8). This will allow us to just check conditions at certain points rather than every point of the membership functions.*

**Remark 9** *As Theorem 3 involves the information of the membership functions in control design, it is an MFD method which is less conservative than the MFI*

method. While Theorem 3 could be reduced to the basic stability conditions using the MFI method for control design.

### 4.3 Simulation Example

To show the effectiveness and efficiency of the proposed theoretical results, the following simulations are performed.

Consider a three-rule IT2 T-S fuzzy model in the form of (4.2) with

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 28.82 & -0.039 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 26.71 & -0.037 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ 22.07 & -0.033 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 & -3.27 \end{bmatrix}^T, \mathbf{B}_2 = \begin{bmatrix} 0 & -2.90 \end{bmatrix}^T, \mathbf{B}_3 = \begin{bmatrix} 0 & -2.09 \end{bmatrix}^T. \mathbf{x} = [x_1 \ x_2]^T.$$

$u_{1H} = -u_{1L} = 5, \epsilon = 0.5.$

The membership functions for the plant (4.2) are chosen as  $\tilde{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+\eta(t))})$ ,  $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ ,  $\tilde{w}_3(x_1) = 1/(1 + e^{-(x_1-4-\eta(t))})$ . Due to parameter uncertainty  $\eta(t) \in [-0.1, 0.1]$ , the membership functions are uncertain grades of membership. The lower and upper membership functions to be approximated by the piecewise linear membership functions for the three-rule plant (4.2) are chosen as  $\underline{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+0.25)})$ ,  $\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1) - \underline{w}_3(x_1)$ ,  $\underline{w}_3(x_1) = 1/(1 + e^{-(x_1-4-0.25)})$ ,  $\bar{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4-0.25)})$ ,  $\underline{w}_2(x_1) = 1 - \bar{w}_1(x_1) - \bar{w}_3(x_1)$ ,  $\bar{w}_3(x_1) = 1/(1 + e^{-(x_1-4+0.25)})$ . The lower and upper membership functions to be approximated by the piecewise linear membership functions for the two-rule controller (4.8) are chosen as  $\underline{m}_1(x_1) = 1 - 1/e^{-(x_1+0.25)/2}$ ,  $\bar{m}_1(x_1) = 1 - 1/e^{-(x_1-0.25)/2}$ ,  $\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$ ,  $\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$ . From (2.14), we can get  $\tilde{m}_j(x_1)$  by setting  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ .

The stability conditions in Theorem 3 are employed to provide the feedback gains. We consider the grades of membership are capped and focus on the region  $x_1 \in [-10, 10]$ . The sample points of  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$  are set as  $x_1 = \{-10, -9, \dots, 9, 10\}$ . Fig. 4.2 shows the membership function information of  $\tilde{h}_{ij}$  with solid lines and  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$  with dash-dot lines. We choose  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 0.001$ . The solution could be found by using third-party MATLAB toolbox SOSTOOLS [15].

With the above settings, the controller gains can be obtained as follows  $\mathbf{K}_1 = \begin{bmatrix} 47.14 & 9.17 \end{bmatrix}$ ,  $\mathbf{K}_2 = \begin{bmatrix} 48.11 & 9.09 \end{bmatrix}$  and  $\mathbf{P} = \begin{bmatrix} 29.93 & 6.36 \\ 6.36 & 1.89 \end{bmatrix}$ . Fig. 4.3 gives the state response of the closed-loop control system which is asymptotically stable with the initial state  $\mathbf{x}(0) = [0.2, -0.1]^T$ . Fig. 4.4 shows the constrained control input with the initial state  $\mathbf{x}(0) = [0.2, -0.1]^T$ . Fig. 4.5 shows the contractively invariant ellipsoid  $\mathbf{\Lambda}_1$  and the trajectories of different initial states.

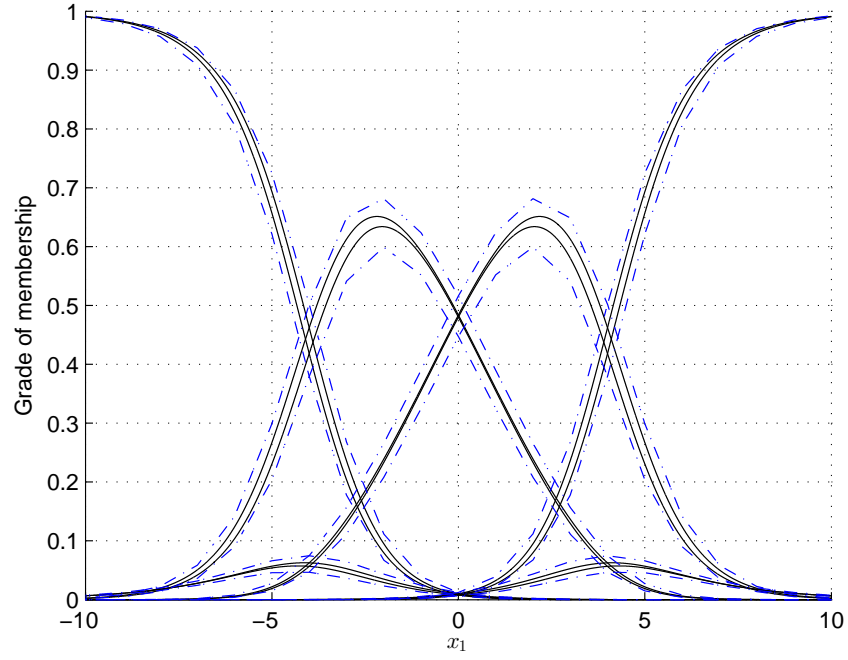


Figure 4.2: Membership function information with three-rule model and two-rule controller

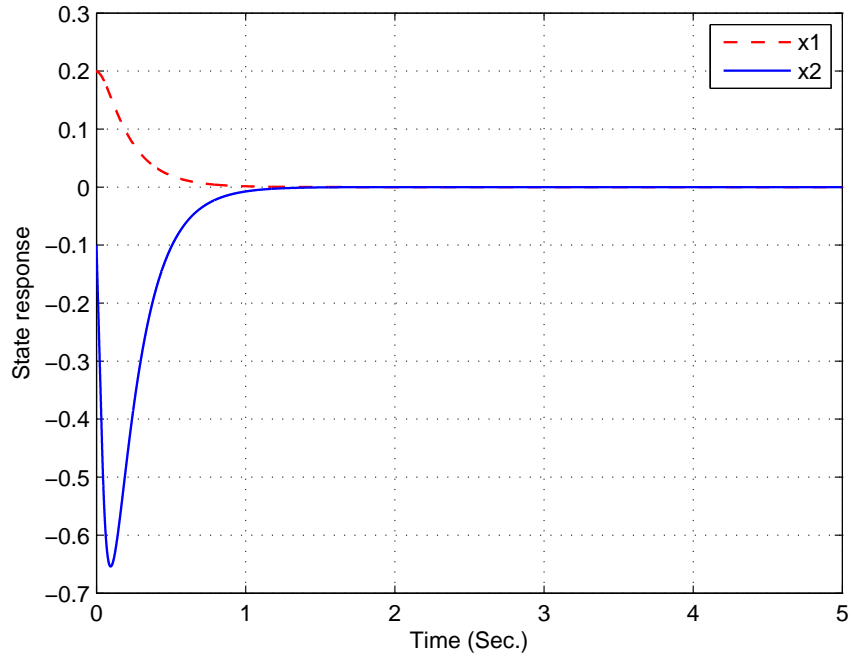


Figure 4.3: State response of the closed-loop control system with  $\mathbf{x}(0) = [0.2, -0.1]^T$

## 4.4 Conclusion

The stability of IT2 T-S FMB control systems with actuator saturation is investigated in this chapter. We have proposed a saturated IT2 T-S fuzzy state-feedback

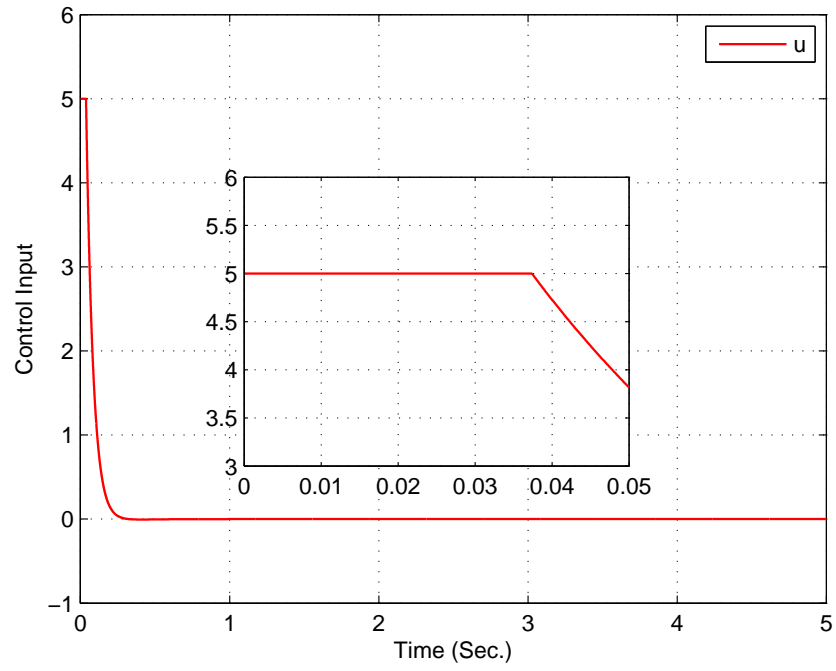


Figure 4.4: Constrained control input of the closed-loop control system with  $\mathbf{x}(0) = [0.2, -0.1]^T$

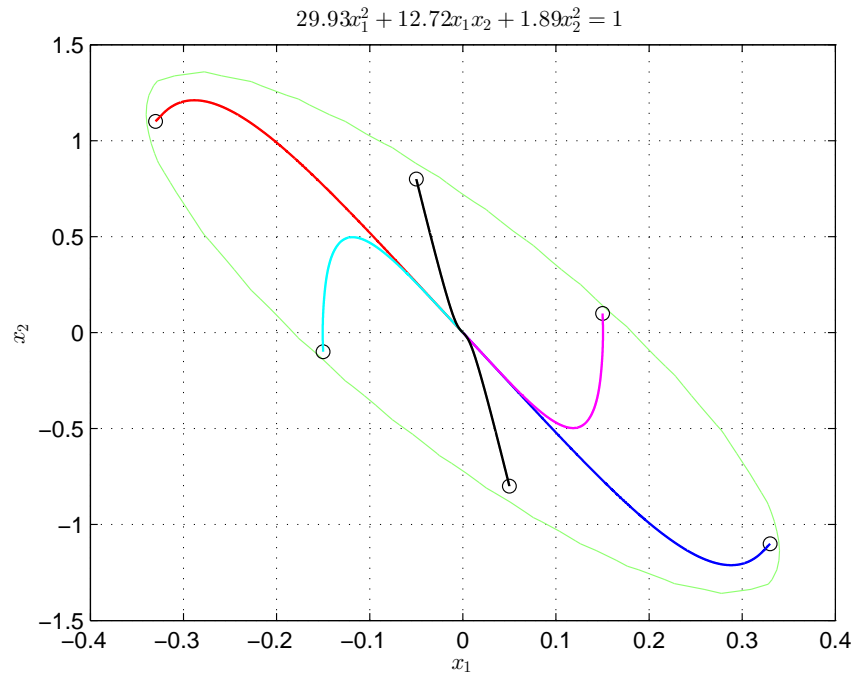


Figure 4.5: State trajectories of the closed-loop control system with different initial states and the contractively invariant ellipsoid  $\Lambda_1$

controller to ensure the asymptotic stability of the closed-loop control system under imperfect premise matching. More design flexibility could be achieved, because it is not required that the fuzzy controller and fuzzy plant have the same premise membership function and/or number of fuzzy rules. The stability conditions based on the invariant ellipsoid come in SOS form and include the information of the membership functions to be more relaxed than MFI method. A numerical example is presented to show the effectiveness of the proposed approach.

# Chapter 5

## Control Design of IT2 PFMB Systems with Time-Varying Delay

In this chapter, the problems of stabilization for IT2 polynomial fuzzy systems with time-varying delay and parameter uncertainties are investigated as a local case. The objective is to design a state-feedback IT2 polynomial fuzzy controller such that the closed-loop control system is asymptotically stable. The conditions for the existence of such a controller are delay dependent and membership function dependent in terms of SOS. Based on a basic lemma to deal with the delay terms, we formulate and solve the problem with more flexibility due to imperfect premise matching that the number of rules and premise membership functions are not necessary the same between the IT2 polynomial fuzzy model and IT2 polynomial fuzzy controller. Piecewise linear membership functions approximations enclosing the original lower and upper membership functions are employed to facilitate the stability analysis. A numerical example indicates the effectiveness of the derived results.

### 5.1 Problem Formulation and Preliminaries

In this section, we will provide the preliminaries in support of the stability analysis and design of the control strategy. The IT2 PFMB control system is shown in Fig. 5.1. An IT2 polynomial fuzzy model is introduced to represent the nonlinear plant with time delay and parameter uncertainties. An IT2 polynomial fuzzy controller is then introduced to control the nonlinear plant by closing the feedback loop. The control problem is formulated as a stabilization problem, which will be investigated based on the Lyapunov approach. To facilitate the stability analysis, the concept of piecewise linear membership functions [36] is recalled which is to represent the lower and upper membership functions in a favorable form. The piecewise linear membership functions approximations are able to approximate the original membership functions and bring the information of the membership functions to the stability analysis. Due to the characteristic of the piecewise linear membership functions,



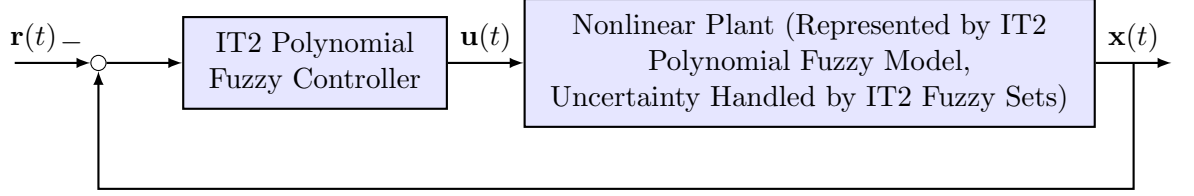


Figure 5.1: A block diagram of IT2 PFMB control system.

the stability of the system can be guaranteed by checking the system stability conditions at some sample points.

### 5.1.1 IT2 Polynomial Fuzzy Model with Time-Varying Delay

Consider a nonlinear system with time-varying delays and parameter uncertainties represented by the following IT2 polynomial fuzzy model with lower and upper membership functions.

**Plant Rule  $i$ :**

IF  $\theta_1(\mathbf{x}(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(\mathbf{x}(t))$  is  $\tilde{M}_{i2} \cdots$  and  $\theta_\Psi(\mathbf{x}(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t))\mathbf{x}(t - d(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{u}(t) \\ \mathbf{x}(t) = \varphi(t), \quad t = [-\bar{d}, 0), \end{cases} \quad (5.1)$$

where  $\tilde{M}_{i\alpha}$  is an IT2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector,  $\mathbf{u}(t) \in \mathbb{R}^l$  is the input vector,  $d(t)$  is the time-varying delay and satisfies  $d(t) \in (0, \bar{d}]$ ,  $\dot{d}(t) \leq m$ ,  $\bar{d}$  and  $m$  are known positive numbers,  $\varphi(t)$  is the initial sequence.  $\mathbf{A}_i(\mathbf{x}(t)) \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B}_i(\mathbf{x}(t)) \in \mathbb{R}^{n \times l}$ ,  $\mathbf{A}_{di}(\mathbf{x}(t)) \in \mathbb{R}^{n \times n}$  are known polynomial matrices as system matrices, input matrices and delayed-state matrices, respectively. The inferred IT2 polynomial fuzzy model is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) (\mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t))\mathbf{x}(t - d(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{u}(t)), \quad (5.2)$$

where (2.7) and (2.8) hold, in which  $\underline{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(\mathbf{x}(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(\mathbf{x}(t)) + \bar{\alpha}_i(\mathbf{x}(t)) = 1$ .

### 5.1.2 IT2 Polynomial Fuzzy Controller

An IT2 polynomial fuzzy controller of  $c$  rules is employed to control the nonlinear plant subject to time delay and parameter uncertainties represented by the IT2 polynomial fuzzy model (5.2), where the  $j$ -th control rule is of the following format:

**Controller Rule  $j$ :**

IF  $\sigma_1(\mathbf{x}(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(\mathbf{x}(t))$  is  $\tilde{N}_{j2} \cdots$  and  $\sigma_\Omega(\mathbf{x}(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$\mathbf{u}(t) = \mathbf{K}_j(\mathbf{x}(t))\mathbf{x}(t), \quad (5.3)$$

where  $\tilde{N}_{j\beta}$  is an IT2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ ,  $\mathbf{K}_j(\mathbf{x}(t))$  are the polynomial feedback gains to be determined. The inferred IT2 polynomial fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t))\mathbf{x}(t), \quad (5.4)$$

where (2.14) and (2.15) hold, in which  $\underline{\beta}_j(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\beta}_j(\mathbf{x}(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(\mathbf{x}(t)) + \bar{\beta}_j(\mathbf{x}(t)) = 1$ .

**Remark 10** We consider the case that the fuzzy rules and the premise membership functions of the plant and controller could be different, i.e.,  $p \neq c$ ,  $\tilde{w}_i(\mathbf{x}(t)) \neq \tilde{m}_i(\mathbf{x}(t))$ , which is referred as imperfect premise matching. This setting would lead to more design flexibility and lower implementation cost when choosing less rules and simpler membership functions of the controller. In addition, the value of  $\tilde{w}_i(\mathbf{x}(t))$  is uncertain, which is not practical to be implemented in the IT2 fuzzy controller, which suggests that  $\tilde{m}_i(\mathbf{x}(t))$  is used instead.

### 5.1.3 IT2 PFMB Control System

With the plant and controller expressions in (5.2) and (5.4), and the property of  $\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1$ ,  $\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1$ ,  $\sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t)) = 1$ , we can have the IT2 PFMB control systems as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t))((\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t))\mathbf{x}(t - d(t))), \quad (5.5)$$

where  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t))$ . In addition,  $\tilde{h}_{ij}(\mathbf{x}(t))$  could be reconstructed as (3.6) in which  $\underline{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$ ,  $\bar{\gamma}_{ij}(\mathbf{x}(t)) \in [0, 1]$  are functions with the property that  $\underline{\gamma}_{ij}(\mathbf{x}(t)) + \bar{\gamma}_{ij}(\mathbf{x}(t)) = 1$ , and  $\bar{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  are the piecewise linear membership function approximations of the upper and lower bound of  $\tilde{h}_{ij}(\mathbf{x}(t))$  with definitions in (3.7) and (3.8) where  $0 \leq \underline{\delta}_{iji_1i_2 \dots i_nk} \leq \bar{\delta}_{iji_1i_2 \dots i_nk} \leq 1$  are scalars to be determined according to  $\tilde{h}_{ij}(\mathbf{x}(t))$ , where (3.9) holds,  $v_{ri_rk}(x_r(t)) \in [0, 1]$ ,  $i_r = 1, 2$  and  $v_{r1k}(x_r(t)) + v_{r2k}(x_r(t)) = 1$ , otherwise  $v_{ri_rk}(x_r(t)) = 0$ ,  $\mathbf{x}(t) \in \Psi_k$ ,  $\cup_{k=1}^q \Psi_k = \Psi$  is the state space of interest.

**Remark 11** The nonlinearity and uncertainties of the IT2 PFMB control system are considered to be embedded in the IT2 membership functions  $\tilde{h}_{ij}(\mathbf{x}(t))$ . Although  $\tilde{h}_{ij}(\mathbf{x}(t)) := \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t))$ , which is uncertain in value, can be characterized by

the known lower and upper membership functions  $\underline{w}_i(\mathbf{x}(t))$  and  $\overline{w}_i(\mathbf{x}(t))$  from the IT2 polynomial fuzzy model and  $\underline{m}_j(\mathbf{x}(t))$  and  $\overline{m}_j(\mathbf{x}(t))$  from the IT2 polynomial fuzzy controller, these membership functions cannot benefit the stability analysis due to the cross terms generated by them which cannot be handled by PDC-based analysis technique. It is necessary to reconstruct them using membership functions of favorable form to make easy the stability analysis. Piecewise linear membership functions  $\underline{h}_{ij}(\mathbf{x}(t))$  and  $\overline{h}_{ij}(\mathbf{x}(t))$  in (3.6) are to serve this purpose and have to satisfy the condition in (3.9). Referring to (3.6), we would like to enclose the region bounded by  $\underline{h}_{ij}(\mathbf{x}(t))$  and  $\overline{h}_{ij}(\mathbf{x}(t))$  (termed as FOU in type-2 fuzzy sets) by  $\underline{h}_{ij}(\mathbf{x}(t))$  and  $\overline{h}_{ij}(\mathbf{x}(t))$ . Consequently, through interpolation, the FOU can be reconstructed by (3.6).

**Remark 12** With the above definitions, in the further stability analysis, we could use scalars  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\overline{\delta}_{ij i_1 i_2 \dots i_n k}$  to characterize  $\underline{h}_{ij}(\mathbf{x}(t))$  and  $\overline{h}_{ij}(\mathbf{x}(t))$  through  $\prod_{r=1}^n v_{ri_r k}(x_r(t))$  which are independent of  $i$  and  $j$ . In a word, the stability conditions involving the membership function information could be achieved by scalars  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\overline{\delta}_{ij i_1 i_2 \dots i_n k}$ .

## 5.2 Stability Analysis

The stability of the IT2 PFMB control system with time-varying delay (5.5) is investigated in this section based on a Lyapunov-Krasovskii functional candidate. To bring the information of the membership functions into the stability analysis, we will develop SOS-based stability conditions depending on the piecewise linear membership functions. In order to bring the information of time-delay, its upper bounds, and time derivative will be considered in the stability analysis. As a result, the SOS-based stability conditions are membership-function and time-delay dependent. If there exists a feasible solution to the SOS-based stability conditions, an IT2 polynomial fuzzy controller can be obtained which can stabilize the nonlinear plant subject to the prescribed bounds of the parameter uncertainties and time-varying delay under consideration.

For simplification reason, in the following analysis, we denote  $\tilde{w}_i(\mathbf{x}(t))$ ,  $\tilde{m}_j(\mathbf{x}(t))$ ,  $\tilde{h}_{ij}(\mathbf{x}(t))$ ,  $\overline{h}_{ij}(\mathbf{x}(t))$  and  $\underline{h}_{ij}(\mathbf{x}(t))$  as  $\tilde{w}_i$ ,  $\tilde{m}_j$ ,  $\tilde{h}_{ij}$ ,  $\overline{h}_{ij}$  and  $\underline{h}_{ij}$ , respectively.

**Theorem 4** Given a constant  $m$ , positive scalar  $\bar{d}$ , predefined scalars  $\overline{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  defined in (3.7) and (3.8),  $K = \{k_1, k_2, \dots, k_q\}$  which is the set of row numbers that the entries of the entire row of  $\mathbf{B}_i(\mathbf{x})$  and  $\mathbf{A}_{di}(\mathbf{x})$  are all zeros and  $\tilde{\mathbf{x}} = (x_{k_1}, x_{k_2}, \dots, x_{k_q})$ . If there exist invertible polynomial matrices  $\mathbf{X}(\tilde{\mathbf{x}})$ , polynomial matrices  $\overline{\mathbf{Y}}_{ij}(\mathbf{x})$ ,  $\tilde{\mathbf{Q}}(\mathbf{x})$ ,  $\tilde{\mathbf{Z}}(\mathbf{x})$  and  $\tilde{\mathbf{T}}(\mathbf{x})$  and  $\mathbf{N}_j(\mathbf{x})$  of appropriate dimensions such that the following SOS-based conditions hold:

$$\begin{aligned}
v_1^T(\mathbf{X}(\tilde{\mathbf{x}}) - \varepsilon_1(\tilde{\mathbf{x}})\mathbf{X})v_1 & \text{ is SOS;} \\
v_1^T(\tilde{\mathbf{Q}}(\mathbf{x}) - \varepsilon_2(\mathbf{x})\mathbf{I})v_1 & \text{ is SOS;} \\
v_1^T(\tilde{\mathbf{Z}}(\mathbf{x}) - \varepsilon_3(\mathbf{x})\mathbf{I})v_1 & \text{ is SOS;} \\
v_2^T(\bar{\mathbf{Y}}_{ij}(\mathbf{x}) - \varepsilon_4(\mathbf{x})\mathbf{I})v_2 & \text{ is SOS;} \\
v_3^T\left(\begin{bmatrix} \tilde{\mathbf{Z}}(\mathbf{x}) & -\tilde{\mathbf{T}}(\mathbf{x}) \\ * & \tilde{\mathbf{Z}}(\mathbf{x}) \end{bmatrix} - \varepsilon_5(x)\mathbf{I}\right)v_3 & \text{ is SOS;} \\
-v_4^T(\Xi_{ij} - \begin{bmatrix} \bar{\mathbf{Y}}_{ij}(\mathbf{x}) & 0 \\ 0 & 0 \end{bmatrix} + \varepsilon_6(x)\mathbf{I})v_4 & \text{ is SOS;} \\
-v_5^T(\sum_{i=1}^p \sum_{j=1}^c (\delta_{ij i_1 i_2 \dots i_n k} \Xi_{ij} \\
+ (\bar{\delta}_{ij i_1 i_2 \dots i_n k} - \underline{\delta}_{ij i_1 i_2 \dots i_n k}) \begin{bmatrix} \bar{\mathbf{Y}}_{ij}(\mathbf{x}) & 0 \\ 0 & 0 \end{bmatrix} + \varepsilon_7(x)\mathbf{I})v_5 & \text{ is SOS,}
\end{aligned} \tag{5.6}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix}, \tag{5.7}$$

with

$$\begin{aligned}
\Xi_{11} &= \mathbf{A}_i(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}}) + \mathbf{B}_i(\mathbf{x})\mathbf{N}_j(\mathbf{x}) + (\mathbf{A}_i(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}}) + \mathbf{B}_i(\mathbf{x})\mathbf{N}_j(\mathbf{x}))^T \\
&\quad + \tilde{\mathbf{Q}}(\mathbf{x}) - \tilde{\mathbf{Z}}(\mathbf{x})/\bar{d} - \sum_{k \in K} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})}{\partial x_k} \mathbf{A}_i^k(\mathbf{x})\mathbf{x},
\end{aligned} \tag{5.8}$$

$$\Xi_{12} = \mathbf{A}_{di}(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}}) + (\tilde{\mathbf{Z}}(\mathbf{x}) + \tilde{\mathbf{T}}(\mathbf{x}))/\bar{d}, \tag{5.9}$$

$$\Xi_{13} = -\tilde{\mathbf{T}}(\mathbf{x})/\bar{d}, \tag{5.10}$$

$$\Xi_{14} = \sqrt{\bar{d}}(\mathbf{X}(\tilde{\mathbf{x}})\mathbf{A}_i(\mathbf{x})^T + \mathbf{N}_j(\mathbf{x})^T\mathbf{B}_i(\mathbf{x})^T), \tag{5.11}$$

$$\Xi_{22} = (m-1)\tilde{\mathbf{Q}}(\mathbf{x}) - (2\tilde{\mathbf{Z}}(\mathbf{x}) + \tilde{\mathbf{T}}(\mathbf{x}) + \tilde{\mathbf{T}}(\mathbf{x})^T)/\bar{d}, \tag{5.12}$$

$$\Xi_{23} = (\tilde{\mathbf{Z}}(\mathbf{x}) + \tilde{\mathbf{T}}(\mathbf{x}))/\bar{d}, \tag{5.13}$$

$$\Xi_{24} = \sqrt{\bar{d}}\mathbf{X}(\tilde{\mathbf{x}})\mathbf{A}_{di}(\mathbf{x})^T, \tag{5.14}$$

$$\Xi_{33} = -\tilde{\mathbf{Z}}(\mathbf{x})/\bar{d}, \tag{5.15}$$

$$\Xi_{34} = 0, \tag{5.16}$$

$$\Xi_{44} = \tilde{\mathbf{Z}}(\mathbf{x}) - 2\mathbf{X}(\tilde{\mathbf{x}}), \tag{5.17}$$

then the IT2 PFMB control system (5.5) is asymptotically stable. Moreover, the feedback gains of the IT2 polynomial fuzzy controller gains can be obtained by  $\mathbf{K}_j(\mathbf{x}) = \mathbf{N}_j(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}})^{-1}$  for all  $j$ .

**Proof** Consider a candidate of Lyapunov-Krasovskii functional with positive

definite matrices  $\mathbf{X}(\tilde{\mathbf{x}})^{-1}$ ,  $\mathbf{Q}$  and  $\mathbf{Z}$  as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (5.18)$$

$$V_1(t) = \mathbf{x}^T(t) \mathbf{X}(\tilde{\mathbf{x}})^{-1} \mathbf{x}(t), \quad (5.19)$$

$$V_2(t) = \int_{t-d(t)}^t \mathbf{x}^T(s) \mathbf{Q} \mathbf{x}(s) ds, \quad (5.20)$$

$$V_3(t) = \int_{-\bar{d}}^0 \int_{t+\theta}^t \dot{\mathbf{x}}^T(s) \mathbf{Z} \dot{\mathbf{x}}(s) ds d\theta. \quad (5.21)$$

Along the trajectories of the closed-loop control system, the corresponding time derivative of  $V(t)$  is given by

$$\begin{aligned} \dot{V}_1(t) &= 2\mathbf{x}^T(t) \mathbf{X}(\tilde{\mathbf{x}})^{-1} \dot{\mathbf{x}}(t) + \mathbf{x}^T(t) \dot{\mathbf{X}}(\tilde{\mathbf{x}})^{-1} \mathbf{x}(t) \\ &= 2 \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \mathbf{x}^T(t) \\ &\quad \times \mathbf{X}(\tilde{\mathbf{x}})^{-1} ((\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{K}_j(\mathbf{x})) \mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t)) \mathbf{x}(t-d(t))) \\ &\quad - \mathbf{x}^T(t) \mathbf{X}(\tilde{\mathbf{x}})^{-1} \sum_{i=1}^p \sum_{k \in K} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})}{\partial x_k} \mathbf{A}_i^k(\mathbf{x}(t)) \mathbf{x}(t) \mathbf{X}(\tilde{\mathbf{x}})^{-1} \mathbf{x}(t), \\ \dot{V}_2(t) &= \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) - (1 - \dot{d}(t)) \mathbf{x}^T(t-d(t)) \mathbf{Q} \mathbf{x}(t-d(t)) \\ &\leq \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) - (1 - m) \mathbf{x}^T(t-d(t)) \mathbf{Q} \mathbf{x}(t-d(t)), \\ \dot{V}_3(t) &= \bar{d} \dot{\mathbf{x}}^T(t) \mathbf{Z} \dot{\mathbf{x}}(t) - \int_{t-\bar{d}}^t \dot{\mathbf{x}}^T(s) \mathbf{Z} \dot{\mathbf{x}}(s) ds, \end{aligned}$$

where  $\mathbf{A}_i^k(x(t))$  denotes the  $k$ -th row of  $\mathbf{A}_i(\mathbf{x}(t))$ .

By applying Lemma 5,  $\dot{V}_3(t)$  can be expressed as

$$\begin{aligned} \dot{V}_3(t) &\leq \bar{d} \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j \\ &\quad \times ((\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{K}_j(\mathbf{x}(t))) \mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t)) \mathbf{x}(t-d(t)))^T \mathbf{Z} \\ &\quad \times ((\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t)) \mathbf{K}_j(\mathbf{x}(t))) \mathbf{x}(t) + \mathbf{A}_{di}(\mathbf{x}(t)) \mathbf{x}(t-d(t))) \\ &\quad + \frac{1}{\bar{d}} v^T(t) \begin{bmatrix} -\mathbf{Z} & \mathbf{Z} + \mathbf{T} & -\mathbf{T} \\ * & -2\mathbf{Z} - \mathbf{T} - \mathbf{T}^T & \mathbf{Z} + \mathbf{T} \\ * & * & -\mathbf{Z} \end{bmatrix} v(t), \end{aligned}$$

with  $v^T(t) = [\mathbf{x}^T(t) \ \mathbf{x}^T(t-d(t)) \ \mathbf{x}^T(t-\bar{d})]$  and subject to  $\begin{bmatrix} -\mathbf{Z} & \mathbf{T} \\ * & -\mathbf{Z} \end{bmatrix} \leq 0$ .

Rewrite  $\dot{V}(t)$  with  $\mathbf{\Omega}_{ij} = \begin{bmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} & -\mathbf{T}/\bar{d} \\ * & \mathbf{\Lambda}_{22} & (\mathbf{Z} + \mathbf{T})/\bar{d} \\ * & * & -\mathbf{Z}/\bar{d} \end{bmatrix},$

where

$$\begin{aligned} \mathbf{\Lambda}_{11} &= \mathbf{X}(\tilde{\mathbf{x}})^{-1}(\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t))) + (\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t)))^T \mathbf{X}(\tilde{\mathbf{x}})^{-1} \\ &\quad + \mathbf{Q} - \mathbf{Z}/\bar{d} + \bar{d}(\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t)))^T \mathbf{Z}(\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t))) \\ &\quad - \mathbf{X}(\tilde{\mathbf{x}})^{-1} \sum_{i=1}^p \sum_{k \in K} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})}{\partial x_k} \mathbf{A}_i^k(\mathbf{x}(t)) \mathbf{x}(t) \mathbf{X}(\tilde{\mathbf{x}})^{-1}, \\ \mathbf{\Lambda}_{12} &= (\mathbf{Z} + \mathbf{T})/\bar{d} + \mathbf{X}(\tilde{\mathbf{x}})^{-1} \mathbf{A}_{di}(\mathbf{x}(t)) + \bar{d}(\mathbf{A}_i(\mathbf{x}(t)) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{K}_j(\mathbf{x}(t)))^T \mathbf{Z} \mathbf{A}_{di}(\mathbf{x}(t)), \\ \mathbf{\Lambda}_{22} &= (m-1)\mathbf{Q} - (2\mathbf{Z} + \mathbf{T} + \mathbf{T}^T)/\bar{d} + \bar{d} \mathbf{A}_{di}(\mathbf{x}(t))^T \mathbf{Z} \mathbf{A}_{di}(\mathbf{x}(t)). \end{aligned}$$

Then we get

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} v^T(t) \mathbf{\Omega}_{ij} v(t) \\ &= \sum_{i=1}^p \sum_{j=1}^c (\underline{h}_{ij} + (\tilde{h}_{ij} - \underline{h}_{ij})) v^T(t) \mathbf{\Omega}_{ij} v(t) \\ &\leq \sum_{i=1}^p \sum_{j=1}^c \underline{h}_{ij} v^T(t) \mathbf{\Omega}_{ij} v(t) + (\bar{h}_{ij} - \underline{h}_{ij}) v^T(t) \mathbf{Y}_{ij} v(t) \\ &= \sum_{i=1}^p \sum_{j=1}^c v^T(t) (\underline{h}_{ij} \mathbf{\Omega}_{ij} + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{Y}_{ij}) v(t), \end{aligned} \tag{5.22}$$

with

$$\begin{aligned} \mathbf{Y}_{ij} &\geq 0, \\ \mathbf{Y}_{ij} &\geq \mathbf{\Omega}_{ij}. \end{aligned}$$

It is required that  $\dot{V}(t)$  in (5.22) is negative definite for system stability. From (5.22),  $\dot{V}(t) < 0$  can be obtained if the following condition holds:

$$\sum_{i=1}^p \sum_{j=1}^c v^T(t) (\underline{h}_{ij} \mathbf{\Omega}_{ij} + (\bar{h}_{ij} - \underline{h}_{ij}) \mathbf{Y}_{ij}) v(t) < 0. \tag{5.23}$$

Rewriting the above condition (5.23) with the definitions in (3.7) and (3.8), and using the fact that  $\sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) = 1$ , one can get the following condition

$$\sum_{i=1}^p \sum_{j=1}^c v^T(t) (\delta_{ij i_1 i_2 \dots i_n k} \mathbf{\Omega}_{ij} + (\bar{\delta}_{ij i_1 i_2 \dots i_n k} - \delta_{ij i_1 i_2 \dots i_n k}) \mathbf{Y}_{ij}) v(t) < 0. \tag{5.24}$$

By using Schur Complement and congruence transformation, we can get the con-

ditions as stated in the theorem with  $\mathbf{K}_j(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}}) = \mathbf{N}_j(\mathbf{x})$ ,  $\tilde{\mathbf{Q}}(\mathbf{x}) = \mathbf{X}(\tilde{\mathbf{x}})\mathbf{Q}\mathbf{X}(\tilde{\mathbf{x}})$ ,  $\tilde{\mathbf{Z}}(\mathbf{x}) = \mathbf{X}(\tilde{\mathbf{x}})\mathbf{Z}\mathbf{X}(\tilde{\mathbf{x}})$ ,  $\tilde{\mathbf{T}}(\mathbf{x}) = \mathbf{X}(\tilde{\mathbf{x}})\mathbf{T}\mathbf{X}(\tilde{\mathbf{x}})$ ,  $\bar{\mathbf{Y}}_{ij}(x) = \text{diag}\{\mathbf{X}(\tilde{\mathbf{x}}), \mathbf{X}(\tilde{\mathbf{x}}), \mathbf{X}(\tilde{\mathbf{x}})\} \times \mathbf{Y}_{ij} \times \text{diag}\{\mathbf{X}(\tilde{\mathbf{x}}), \mathbf{X}(\tilde{\mathbf{x}}), \mathbf{X}(\tilde{\mathbf{x}})\}$ . After getting  $\mathbf{K}_j(\mathbf{x})$ ,  $\tilde{\mathbf{Q}}(\mathbf{x})$ ,  $\tilde{\mathbf{Z}}(\mathbf{x})$ ,  $\tilde{\mathbf{T}}(\mathbf{x})$  and  $\bar{\mathbf{Y}}_{ij}(\mathbf{x})$ , we need to make sure that the above five equations still hold.  $\mathbf{X}(\tilde{\mathbf{x}})$  is a polynomial matrix of zero degree would be one of the many cases that guarantees the above equations.

**Remark 13**  $K$  is the set of row numbers that the entries of entire row of  $\mathbf{B}_i(\mathbf{x})$  and  $\mathbf{A}_{di}(\mathbf{x})$  are all zero. This statement is needed in the analysis mainly because we want to deal with the term  $\dot{\mathbf{X}}(\tilde{\mathbf{x}})^{-1}$  in  $\dot{V}_1(t)$ . We denote  $K = \{k_1, k_2, \dots, k_q\}$  which is the set of row numbers that the entries of the entire row of  $\mathbf{B}_i(\mathbf{x})$  and  $\mathbf{A}_{di}(\mathbf{x})$  are all zeros and  $\tilde{\mathbf{x}} = (x_{k_1}, x_{k_2}, \dots, x_{k_q})$  with  $\sum_{k \notin K} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})^{-1}}{\partial x_k} = 0$ . Then we have  $\dot{\mathbf{X}}(\tilde{\mathbf{x}})^{-1} = \sum_{k \in K} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})^{-1}}{\partial x_k} \sum_{i=1}^p [\mathbf{A}_i^k(\mathbf{x})\mathbf{x} + \mathbf{A}_{di}^k(\mathbf{x})\mathbf{x}(t-d(t)) + \mathbf{B}_i^k(\mathbf{x})\mathbf{u}]$ , where  $\mathbf{A}_i^k(\mathbf{x})$ ,  $\mathbf{A}_{di}^k(\mathbf{x})$  and  $\mathbf{B}_i^k(\mathbf{x})$  denote the  $k$ th row of  $\mathbf{A}_i(\mathbf{x})$ ,  $\mathbf{A}_{di}(\mathbf{x})$  and  $\mathbf{B}_i(\mathbf{x})$ , respectively. With the above setting, we can cancel the terms  $\mathbf{A}_{di}^k(\mathbf{x})\mathbf{x}(t-d(t))$  and  $\mathbf{B}_i^k(\mathbf{x})\mathbf{u}$ . Based on the property that  $\frac{\partial \mathbf{X}(\tilde{\mathbf{x}})^{-1}}{\partial x_k} = -\mathbf{X}(\tilde{\mathbf{x}})^{-1} \frac{\partial \mathbf{X}(\tilde{\mathbf{x}})}{\partial x_k} \mathbf{X}(\tilde{\mathbf{x}})^{-1}$ , we can get  $\dot{V}_1(t)$  shown in the proof.

**Remark 14** There is no general way to find  $\mathbf{X}(\tilde{\mathbf{x}})$ . If it is unknown, then the controller is not available. So we choose  $\mathbf{X}(\tilde{\mathbf{x}})$  as a polynomial matrix of zero degree as a special case that guarantees the above equations.

**Remark 15** The conditions in (5.23) and (5.24) are equivalent. In (5.23), the condition is in terms of the lower and upper membership functions  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$ , which are continuous functions so that the number of stability conditions in (5.23) is actually infinite, which is impractical to be solved numerically. By rewriting (5.23) in (5.24), the stability condition is no longer depending on the lower and upper membership functions  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$  but their sample points  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$ . Due to the advantage of piecewise linear membership functions, satisfying the stability condition in (5.23) is implied by satisfying the conditions in (5.24), which requires the checking only at the sample points so that the number of stability conditions become finite. It could be noted that dividing the region of  $x$  into more partitions could further reduce the conservatism. The more local upper and lower bounds of the membership functions involved in could lead to more relaxed results while the computational burden would be heavier.

As Theorem 4 involves the information of the membership functions in control design, it is an MFD method. By removing the membership function information in the analysis, we can get the following MFI theorem.

**Theorem 5** *Given a constant  $m$ , positive scalar  $\bar{d}$ , if there exist polynomial matrices  $\mathbf{X}(\tilde{\mathbf{x}})$ ,  $\tilde{\mathbf{Q}}(\mathbf{x})$ ,  $\tilde{\mathbf{Z}}(\mathbf{x})$ ,  $\tilde{\mathbf{T}}(\mathbf{x})$  and  $\mathbf{N}_j(\mathbf{x})$  of appropriate dimensions such that the following SOS-based conditions hold:*

$$\begin{aligned}
v_1^T(\mathbf{X}(\tilde{\mathbf{x}}) - \varepsilon_1(\tilde{\mathbf{x}})\mathbf{I})v_1 & \text{ is SOS;} \\
v_1^T(\tilde{\mathbf{Q}}(\mathbf{x}) - \varepsilon_2(\mathbf{x})\mathbf{I})v_1 & \text{ is SOS;} \\
v_1^T(\tilde{\mathbf{Z}}(\mathbf{x}) - \varepsilon_3(\mathbf{x})\mathbf{I})v_1 & \text{ is SOS;} \\
v_3^T\left(\begin{bmatrix} \tilde{\mathbf{Z}}(\mathbf{x}) & -\tilde{\mathbf{T}}(\mathbf{x}) \\ * & \tilde{\mathbf{Z}}(\mathbf{x}) \end{bmatrix} - \varepsilon_5(\mathbf{x})\mathbf{I}\right)v_3 & \text{ is SOS;} \\
-v_4^T(\Xi_{ij} + \varepsilon_6(\mathbf{x})\mathbf{I})v_4 & \text{ is SOS,}
\end{aligned} \tag{5.25}$$

the definitions of all the variables are given in Theorem 4. Then the IT2 PFMB control system (5.5) is asymptotically stable. Moreover, the feedback gains of the IT2 polynomial fuzzy controller can be obtained by  $\mathbf{K}_j(\mathbf{x}) = \mathbf{N}_j(\mathbf{x})\mathbf{X}(\tilde{\mathbf{x}})^{-1}$  for all  $j$ .

**Remark 16** *Compared to Theorem 4, Theorem 5 removes conditions involving membership function information and slack matrices. From this point of view, Theorem 5 is then more conservative than Theorem 4, which will be testified in next section. While Theorem 5 is not as complicated as Theorem 4 and would consume less time to get a solution.*

**Remark 17** *The Lyapunov-Krasovskii functional we choose for Theorems 4 and 5 consists of three parts:  $V_1(t)$ ,  $V_2(t)$  and  $V_3(t)$ .  $V_1(t)$  is a quadratic functional commonly used in stability analysis.  $V_2(t)$  and  $V_3(t)$  are chosen to include the delay information (bound of the delay and the derivative of delay) in the analysis. Both theorems could be modified to tackle systems without time-varying delay by removing  $V_2(t)$  and  $V_3(t)$  in  $V(t)$  following the similar derivation.*

## 5.3 Simulation Example

In this section, a numerical example is presented to demonstrate the potential and validity of our developed theoretical results. For a time-delayed nonlinear system represented by an IT2 polynomial fuzzy model, Theorem 4 and Theorem 5 are employed to design an IT2 polynomial fuzzy controller to ensure the stability of the IT2 PFMB system. Moreover, we want to show that our MFD approach (Theorem 4) is less conservative than MFI approach (Theorem 5). Details of membership function information, feedback gains of the controller, and the state response and control input of the system could be found below.

Consider a nonlinear plant subject to parameter uncertainties and time delay represented by a three-rule IT2 polynomial fuzzy model in the form of (5.2) with  $\mathbf{A}_1(\mathbf{x}) = \begin{bmatrix} 0 & -x_1 + 1 \\ 1 & 2 \end{bmatrix}$ ,  $\mathbf{A}_2(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & -0.01x_1^2 + 2 \end{bmatrix}$ ,  $\mathbf{A}_3(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & -0.01x_1 \end{bmatrix}$ ,



$$\mathbf{B}_1(\mathbf{x}) = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^T, \mathbf{B}_2(\mathbf{x}) = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^T, \mathbf{B}_3(\mathbf{x}) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \\ \mathbf{A}_{d1}(\mathbf{x}) = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \mathbf{A}_{d2}(\mathbf{x}) = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.1 \end{bmatrix}, \mathbf{A}_{d3}(\mathbf{x}) = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \\ \mathbf{x} = [x_1 \ x_2]^T, \varphi(t) = 0 \text{ when } t \in [-\bar{d}, 0).$$

The membership functions for the plant (5.2) are chosen as  $\tilde{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+\eta(t))})$ ,  $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ ,  $\tilde{w}_3(x_1) = 1/(1 + e^{-(x_1-4-\eta(t))})$ . Due to parameter uncertainty  $\eta(t) \in [-0.1, 0.1]$ , the grades of membership function becomes uncertain in value.

The lower and upper membership functions to be approximated by the piecewise linear membership functions for the three-rule plant (5.2) are chosen as  $\underline{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+d_1)})$ ,  $\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1) - \underline{w}_3(x_1)$ ,  $\underline{w}_3(x_1) = 1/(1 + e^{-(x_1-4-d_1)})$ ,  $\bar{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4-d_1)})$ ,  $\underline{w}_2(x_1) = 1 - \bar{w}_1(x_1) - \bar{w}_3(x_1)$ ,  $\bar{w}_3(x_1) = 1/(1 + e^{-(x_1-4+d_1)})$ .

A two-rule IT2 polynomial fuzzy controller in the form of (5.4) is employed to control the nonlinear plant. The lower and upper membership functions are chosen as  $\underline{m}_1(x_1) = 1 - 1/e^{-(x_1+d_2)/2}$ ,  $\bar{m}_1(x_1) = 1 - 1/e^{-(x_1-d_2)/2}$ ,  $\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$ ,  $\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$ . From (2.14), we can get  $\tilde{m}_j(x_1)$  by setting  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ .

We consider region of interest to be  $x_1 \in [-10, 10]$  and the membership grades outside this region are capped. The sample points of  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$  are set as Case 1:  $x_1 = \{-10, -8, \dots, 8, 10\}$ , Case 2:  $x_1 = \{-10, -9, \dots, 9, 10\}$  and Case 3:  $x_1 = \{-10, -9.5, \dots, 9.5, 10\}$ , respectively. To determine  $\bar{h}_{ij}(x_1)$  and  $\underline{h}_{ij}(x_1)$ , we consider Case 2 for demonstration purposes.  $\Psi_k : \underline{x}_{1,k} \leq x_1 \leq \bar{x}_{1,k}$ , where  $\underline{x}_{1,k} = k - 11$  and  $\bar{x}_{1,k} = k - 10$ ,  $k = 1, 2, \dots, 19, 20$ . By choosing  $v_{11k}(x_1) = 1 - (x_1 - \underline{x}_{1,k})/(\bar{x}_{1,k} - \underline{x}_{1,k})$  and  $v_{12k}(x_1) = 1 - v_{11k}(x_1)$ , and the scalars  $\bar{\delta}_{ij1k} = \bar{w}_i(\underline{x}_{1,k})\bar{m}_j(\underline{x}_{1,k})$ ,  $\bar{\delta}_{ij2k} = \bar{w}_i(\bar{x}_{1,k})\bar{m}_j(\bar{x}_{1,k})$ ,  $\underline{\delta}_{ij1k} = \underline{w}_i(\underline{x}_{1,k})\underline{m}_j(\underline{x}_{1,k})$  and  $\underline{\delta}_{ij2k} = \underline{w}_i(\bar{x}_{1,k})\underline{m}_j(\bar{x}_{1,k})$  for all  $k$ , we can define  $\bar{h}_{ij}(x_1)$  and  $\underline{h}_{ij}(x_1)$  as  $\bar{h}_{ij}(x(t)) = \sum_{k=1}^{20} (v_{11k}\bar{\delta}_{ij1k} + v_{12k}\bar{\delta}_{ij2k})$  and  $\underline{h}_{ij}(x(t)) = \sum_{k=1}^{20} (v_{11k}\underline{\delta}_{ij1k} + v_{12k}\underline{\delta}_{ij2k})$ . The same idea applies to Cases 1 and 3 to determine  $\bar{h}_{ij}(x_1)$  and  $\underline{h}_{ij}(x_1)$ .

Figs. 5.2-5.4 show the membership function information of  $\tilde{h}_{ij}$ ,  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$  for Cases 1-3, respectively. Table 5.1 presents the parameter values of  $d_1$  and  $d_2$  for the three cases. The introduction of  $d_1$  and  $d_2$  is for the purpose of obtaining fitter membership function approximations for different sampling intervals.

Table 5.1: Parameter values for sampling interval,  $d_1$  and  $d_2$  for Cases 1-3

	Case 1	Case 2	Case 3
sampling interval	2.0000	1.0000	0.5000
$d_1$	0.7978	0.2461	0.1384
$d_2$	0.6099	0.2499	0.0600

**Remark 18** As one can see from Figs. 5.2-5.4, the smaller sampling interval for  $x_1$  could lead the approximated membership functions closer to the original ones. For

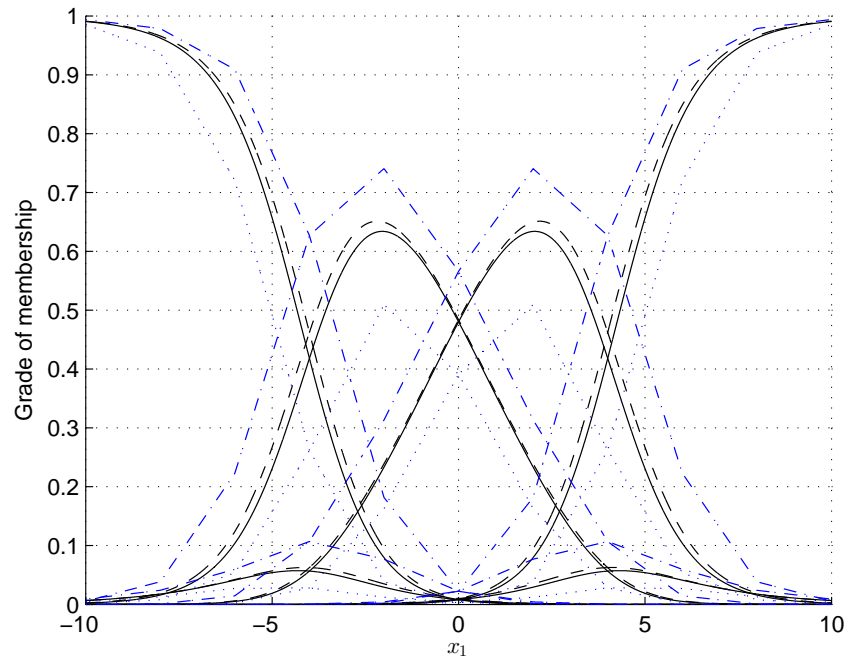


Figure 5.2: Membership function information with three-rule model and two-rule controller for Case 1 with  $x_1 = \{-10, -8, \dots, 8, 10\}$ ,  $d_1 = 0.7978$  and  $d_2 = 0.6099$ . Solid lines are for the original lower membership functions and dashed lines are for the original upper membership functions. Dotted lines are for the approximated lower piecewise linear membership functions and dashed-dot lines are for the approximated upper piecewise linear membership functions.

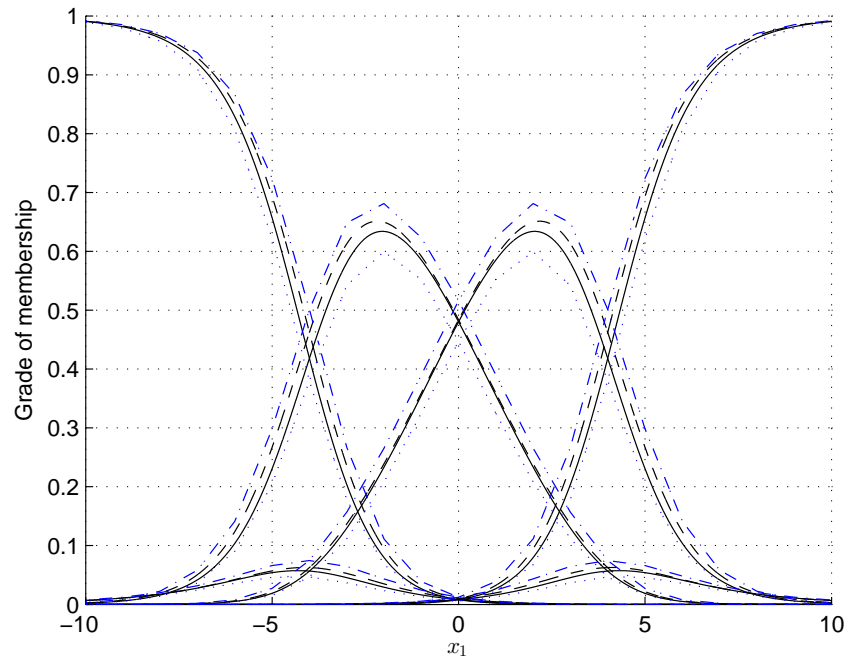


Figure 5.3: Membership function information with three-rule model and two-rule controller for Case 2 with  $x_1 = \{-10, -9, \dots, 9, 10\}$ ,  $d_1 = 0.2461$  and  $d_2 = 0.2499$ . Solid lines are for the original lower membership functions and dashed lines are for the original upper membership functions. Dotted lines are for the approximated lower piecewise linear membership functions and dashed-dot lines are for the approximated upper piecewise linear membership functions.

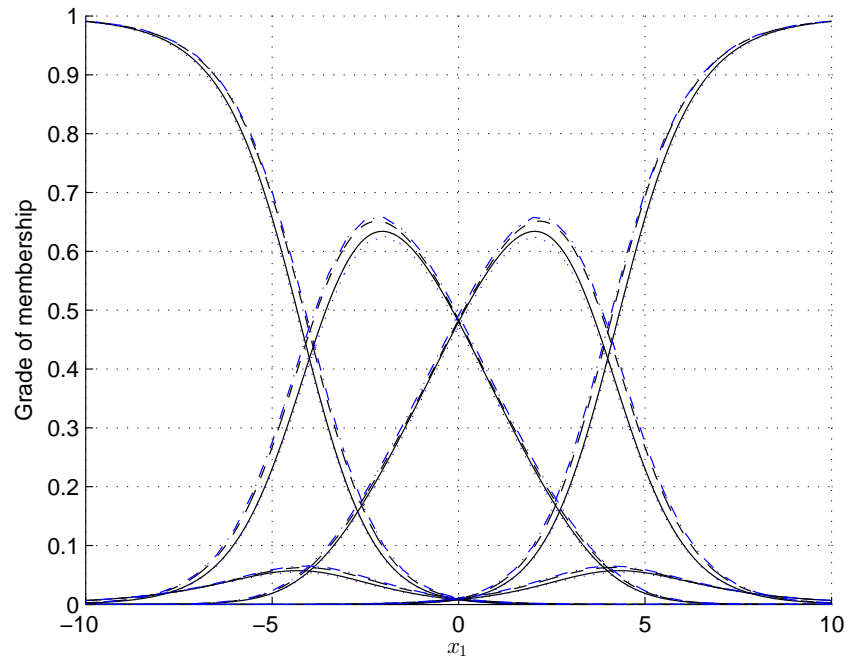


Figure 5.4: Membership function information with three-rule model and two-rule controller for Case 3 with  $x_1 = \{-10, -9.5, \dots, 9.5, 10\}$ ,  $d_1 = 0.1384$  and  $d_2 = 0.0600$ . Solid lines are for the original lower membership functions and dashed lines are for the original upper membership functions. Dotted lines are for the approximated lower piecewise linear membership functions and dashed-dot lines are for the approximated upper piecewise linear membership functions.

each case, one could choose even larger values of  $d_1$  and  $d_2$  to include the information of the membership functions into the stability analysis, while this is not recommended as larger  $d_1$  and  $d_2$  would increase the conservatism. The values of  $d_1$  and  $d_2$  listed in Table 5.1 are the smallest values for each case making sure the original membership functions are enclosed by the approximated piecewise linear membership functions.

For Cases 1-3, the stability conditions in Theorem 4 and Theorem 5 are employed to stabilize the nonlinear plant. We choose  $\varepsilon_1(\tilde{\mathbf{x}}) = \varepsilon_2(\mathbf{x}) = \varepsilon_3(\mathbf{x}) = \varepsilon_4(\mathbf{x}) = \varepsilon_5(\mathbf{x}) = \varepsilon_6(\mathbf{x}) = \varepsilon_7(\mathbf{x}) = 0.001$ . The solution could be found by using the third-party MATLAB toolbox SOSTOOLS [15]. Table 5.2 summarizes the maximum allowable delay bound for the polynomial system with feasible solutions by Theorem 4 and Theorem 5 for Case 2. The reason that Case 2 is used to present the simulation results is a compromise between better approximated membership functions (smaller sampling interval) and lower computational burden (larger sampling interval). It could be seen from Table 5.2 that for the same  $m$  (upper bound of the delay derivative) larger maximum delay bound could be obtained by Theorem 4 compared to Theorem 5, which means Theorem 4 can still stabilize the system when Theorem 5 fails to do so for a system with larger time delay. In this sense, our MFD approach (Theorem 4) presents less conservative results compared to the MFI approach (Theorem 5).

Table 5.2: Comparison of maximum delay bound for different values of  $m$

Considered results	$m = 0.0010$	$m = 1.0000$
Theorem 4	0.0869s	0.0795s
Theorem 5	0.0215s	0.0175s

With the above settings and  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ , for  $m = 0.9\bar{d} = 0.0010$ , the feedback gains obtained by Theorem 4 are

$$\mathbf{K}_1(x_1) = \begin{bmatrix} -3.1755 \times 10^{-5}x_1^2 + 4.9220 \times 10^{-3}x_1 - 0.6202 \\ -2.9771 \times 10^{-2}x_1^2 + 0.1319x_1 - 50.4205 \end{bmatrix}^T,$$

$$\mathbf{K}_2(x_1) = \begin{bmatrix} -9.9620 \times 10^{-5}x_1^2 - 6.0112 \times 10^{-5}x_1 - 0.3184 \\ -1.0688 \times 10^{-2}x_1^2 - 9.3118 \times 10^{-3}x_1 - 49.8524 \end{bmatrix}^T$$

with polynomial  $\mathbf{X}(\tilde{\mathbf{x}}) = \begin{bmatrix} 12.3052 & 0.0868 \\ 0.0868 & 0.2004 \end{bmatrix}$ ; for  $m = 0.9\bar{d} = 0.0100$ , the feedback gains obtained by Theorem 4 are listed as follows:

$$\mathbf{K}_1(x_1) = \begin{bmatrix} -1.0764 \times 10^{-5}x_1^2 + 1.2492 \times 10^{-3}x_1 - 0.8623 \\ -9.8473 \times 10^{-3}x_1^2 + 0.1177x_1 - 82.9392 \end{bmatrix}^T,$$

$$\mathbf{K}_2(x_1) = \begin{bmatrix} -1.9279 \times 10^{-5}x_1^2 + 8.6310 \times 10^{-5}x_1 - 0.6785 \\ -4.8110 \times 10^{-3}x_1^2 - 6.5791 \times 10^{-3}x_1 - 77.3896 \end{bmatrix}^T$$

with polynomial  $\mathbf{X}(\tilde{\mathbf{x}}) = \begin{bmatrix} 8.6423 & 0.0163 \\ 0.0163 & 0.0514 \end{bmatrix}$ .

Figs. 5.5-5.6 give the state response of the closed-loop control system which is asymptotically stable with the initial state  $\mathbf{x}(0) = [-10, -10]^T$  with  $m = 0.0010$

and  $m = 0.0100$ , respectively. Figs. 5.7-5.8 show the control input of the closed-loop control system with the initial state  $\mathbf{x}(0) = [-10, -10]^T$  with  $m = 0.0010$  and  $m = 0.0100$ , respectively. As one can see from the figures, our theorem is able to stabilize the system for different values of the delay bound.

**Remark 19** *In the simulation, we set  $\mathbf{X}(\tilde{\mathbf{x}})$ ,  $\tilde{\mathbf{Q}}(\mathbf{x})$ ,  $\tilde{\mathbf{Z}}(\mathbf{x})$  and  $\tilde{\mathbf{T}}(\mathbf{x})$  as polynomials of degree 0, and set  $\mathbf{N}_j(x)$  and  $\bar{\mathbf{Y}}_{ij}(\mathbf{x})$  as polynomials with monomials in  $x_1$  of degree 2. The degrees of the polynomials could be modified according to the users. Normally, the higher order would result in less conservative results and longer computational time.*

**Remark 20** *The software we use for numerical simulation are listed as follows: MATLAB R2012b, SOSTOOLS v3.0 and SeDuMi v1.3.*

**Remark 21** *Details of comparison between T-S FMB system and PFMB system with this numerical example could be found in Appendix A.*

**Remark 22** *The design process of the proposed method would start from considering a practical nonlinear system and then representing it by an IT2 polynomial fuzzy model. We then choose appropriate fuzzy rules and membership functions for the IT2 polynomial fuzzy controller based on the needs. After applying the theorems developed in the chapter, we can get the gains of the controller and finally implement the controller to the nonlinear system.*

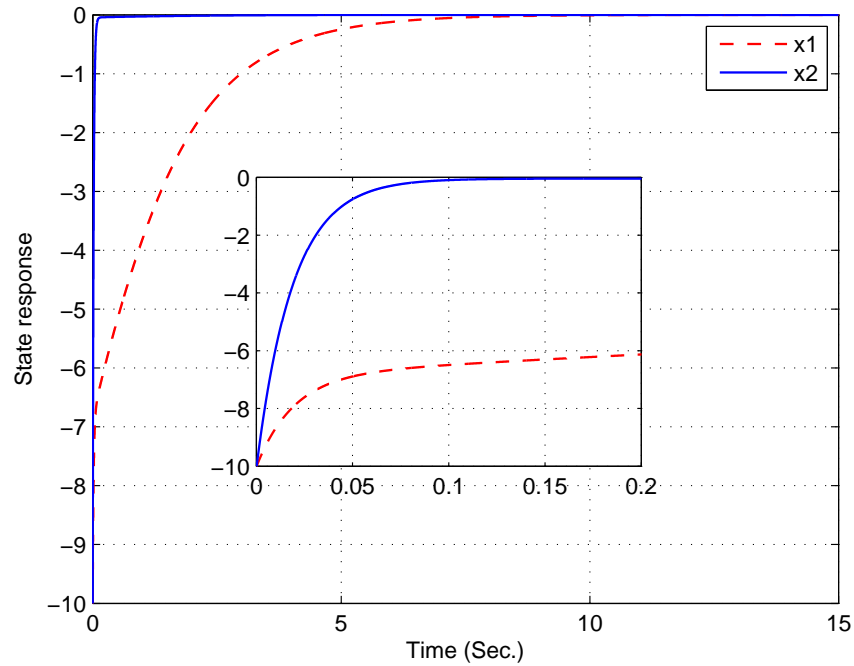


Figure 5.5: State response of the closed-loop control system with  $\mathbf{x}(0) = [-10 \ -10]^T$ ,  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ ,  $m = 0.9\bar{d} = 0.0010$ .

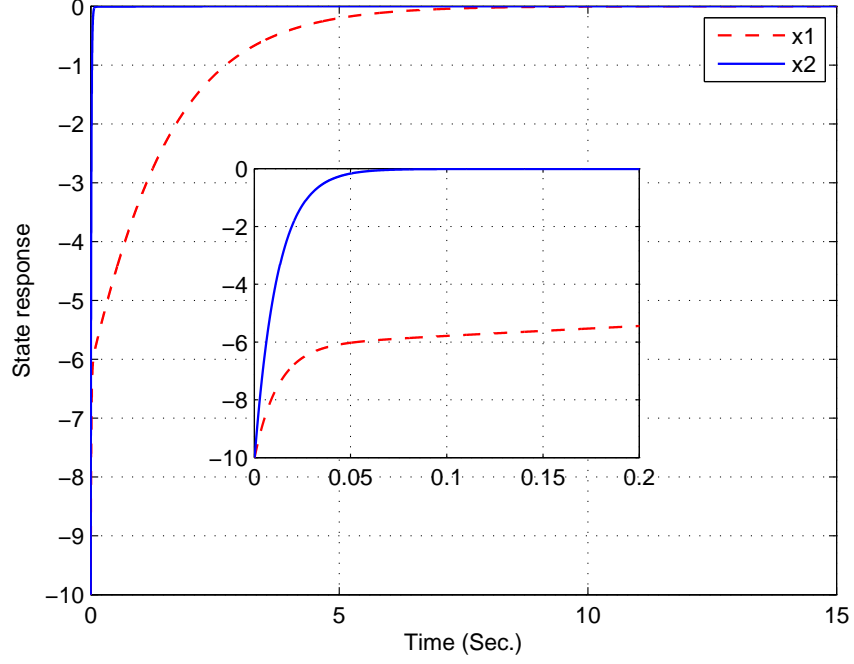


Figure 5.6: State response of the closed-loop control system with  $\mathbf{x}(0) = [-10 \ -10]^T$ ,  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ ,  $m = 0.9\bar{d} = 0.0100$ .

## 5.4 Conclusion

The stability of IT2 PFMB control systems with time-varying delay and parameter uncertainties under imperfect premise matching has been investigated in this chapter. A state-feedback IT2 polynomial fuzzy controller has been proposed to ensure the asymptotic stability of the closed-loop time-delayed systems under IT2 PFMB control framework. To facilitate the MFD stability analysis, piecewise linear membership functions have been employed to approximate the original upper and lower membership functions. More design flexibility and practicality could be achieved, because it is not required that the polynomial fuzzy controller and polynomial fuzzy plant have the same premise membership function and/or number of fuzzy rules. The stability conditions come in SOS form based on a dedicated chosen delay-dependent Lyapunov-Krasovskii functional. A numerical example is presented to show the effectiveness of the proposed approach.

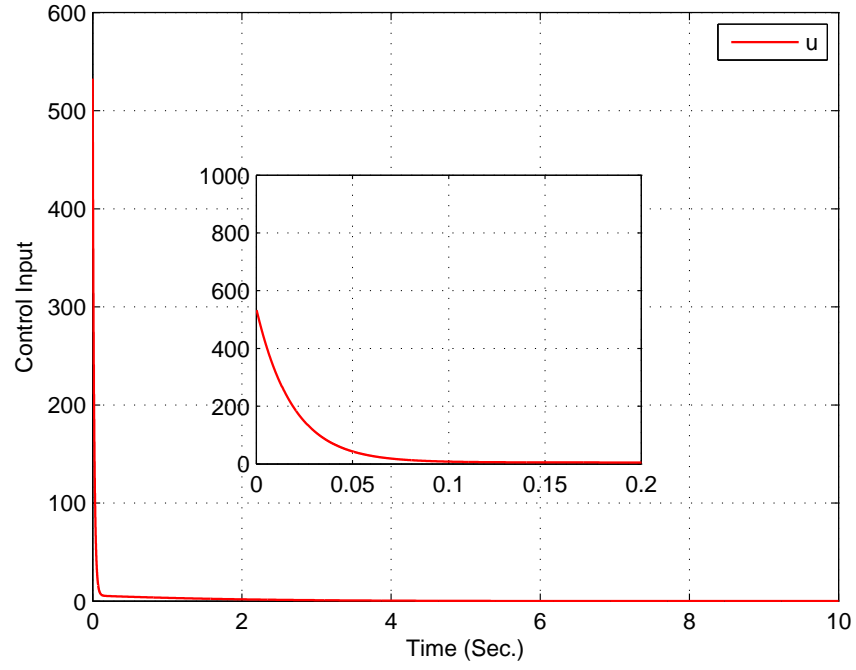


Figure 5.7: Control input of the closed-loop control system with  $\mathbf{x}(0) = [-10 \ -10]^T$ ,  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ ,  $m = 0.9\bar{d} = 0.0010$ .

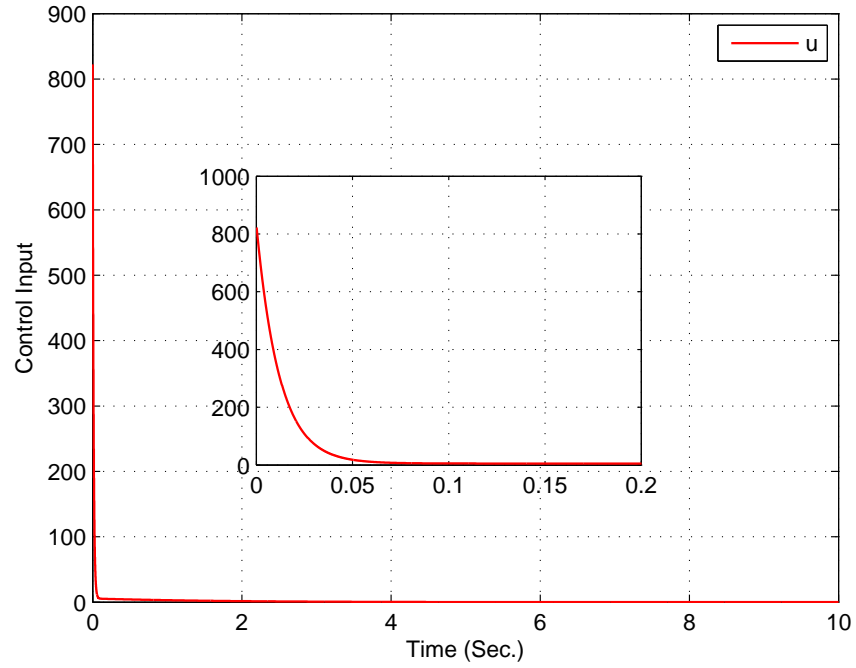


Figure 5.8: Control input of the closed-loop control system with  $\mathbf{x}(0) = [-10 \ -10]^T$ ,  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ ,  $m = 0.9\bar{d} = 0.0100$ .



# Chapter 6

## Conclusion and Future Work

### 6.1 Conclusion

In this thesis, the stability analysis and control synthesis of the IT2 FMB control systems have been investigated. The main contribution is presented in Chapters 3, 4, and 5.

In Chapter 3, stabilization for T-S FMB systems with time-varying delay and parameter uncertainties under IT2 fuzzy logic has been investigated. IT2 T-S fuzzy state feedback controller has been proposed to ensure the stability of the closed-loop system. Both MFI and MFD stability conditions have been established in terms of LMIs. A numerical example is presented to show the effectiveness of the proposed approach. The study in this chapter not only extends the FMB control strategy to system with delay, but also serves as the foundation of the study in Chapter 5.

In Chapter 4, stabilization for T-S FMB systems with actuator saturation and parameter uncertainties under IT2 fuzzy logic has been investigated. Saturated IT2 T-S fuzzy state feedback controller has been proposed to ensure the stability of the closed-loop system. MFD stability conditions have been established in terms of SOS. A numerical example indicates the effectiveness of the derived results. The study in this chapter extends the FMB control strategy to system with actuator saturation.

In Chapter 5, stabilization for PFMB systems with time-varying delay and parameter uncertainties under IT2 fuzzy logic has been investigated. IT2 polynomial fuzzy state feedback controller has been proposed to ensure the stability of the closed-loop system. Both MFI and MFD stability conditions have been established in terms of SOS. A numerical example is presented to show the effectiveness of the proposed approach. This chapter could be regarded as a generalization of Chapter 3 from T-S FMB systems to PFMB systems. To our best knowledge, this could be the first time to study the stabilization problem of time-delayed systems under IT2 PFMB control framework.

All the work in the above chapters are under the concept of imperfect premise matching, where the fuzzy controller and fuzzy plant do not have to share the same

premise membership function and/or number of fuzzy rules. Examples have been presented to show the effectiveness of the proposed approach and less conservativeness of our MFD approach where applicable.

## 6.2 Future Work

The potential research directions are listed as follows:

- 1) In this thesis, for FMB control systems with time-varying delay, we employ Lyapunov functions based on quadratic form. In future work, different/sophisticated Lyapunov functions could be applied to the research, e.g. polynomial Lyapunov functions.
- 2) In this thesis, for FMB control systems with actuator saturation, only T-S FMB control systems are considered. In future work, the generalized problem for PFMB control systems is left to be solved.
- 3) In this thesis, only continuous-time system is investigated. In future work, the discrete-time case could be another direction.
- 4) In this thesis, piecewise linear membership function approximation is adopted in the analysis. In future work, other membership function approximations could be applied for the MFD approach, such as staircase and polynomial membership function approximations. More information from the membership functions would lead to less conservativeness but increase the computational burden. This compromise should be studied further.

# Appendix A

## Comparison between T-S FMB System and PFMB System with Example in Section 5.3

In Section 5.3, the nonlinear plant is represented as a polynomial fuzzy model. Based on that model, a polynomial fuzzy controller is introduced to control the plant by closing the feedback loop. While in this section, we would like to reconstruct the same nonlinear plant as a T-S fuzzy model and compare with the results we presented in Section 5.3.

Recall the same three-rule fuzzy model we use for the numerical example in Section 5.3 with  $\mathbf{A}_1 = \begin{bmatrix} 0 & -x_1 + 1 \\ 1 & 2 \end{bmatrix}$ ,  $\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 1 & -0.01x_1^2 + 2 \end{bmatrix}$ ,  $\mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ 1 & -0.01x_1 \end{bmatrix}$ ,  $\mathbf{B}_1 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^T$ ,  $\mathbf{B}_2 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^T$ ,  $\mathbf{B}_3 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ ,  $\mathbf{A}_{d1} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$ ,  $\mathbf{A}_{d2} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.1 \end{bmatrix}$ ,  $\mathbf{A}_{d3} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}$ ,  $\mathbf{x} = [x_1 \ x_2]^T$ ,  $\varphi(t) = 0$  when  $t \in [-\bar{d}, 0)$ . The membership functions are chosen as  $\tilde{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1 + 4 + \eta(t))})$ ,  $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ ,  $\tilde{w}_3(x_1) = 1/(1 + e^{-(x_1 - 4 - \eta(t))})$ . Due to parameter uncertainty  $\eta(t) \in [-0.1, 0.1]$ , the grades of membership function becomes uncertain in value. With these settings, the model could be rewritten as

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^3 \tilde{w}_i(x_1)(\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))).$$

The above equation can be rewritten as

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & z_1 \\ 1 & z_2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} z_3 \\ 1 \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} z_4 & 0 \\ 0 & z_5 \end{bmatrix} \mathbf{x}(t - d(t)),$$

where  $z_1 = 1 - \tilde{w}_1 x_1$ ,  $z_2 = -0.01\tilde{w}_2 x_1^2 - 0.01\tilde{w}_3 x_1 + 2\tilde{w}_1 + 2\tilde{w}_2$ ,  $z_3 = 0.5 + 0.5\tilde{w}_3$ ,

$z_4 = -0.1 - 0.1\tilde{w}_2$ , and  $z_5 = -0.1 - 0.1\tilde{w}_3$ . As the parameter uncertainty  $\eta(t) \in [-0.1, 0.1]$  and  $x_1 \in [-10, 10]$ , we can calculate the maximum and minimum for  $z_1$  till  $z_5$  as  $z_1 \in [0.9926, 10.9776]$ ,  $z_2 \in [-0.0975, 1.9978]$ ,  $z_3 \in [0.5000, 0.9989]$ ,  $z_4 \in [-0.1967, -0.1002]$ , and  $z_5 \in [-0.1998, -0.1000]$ . By using the sector nonlinearity concept, we can construct the model in T-S fuzzy logic. For the nonlinear term  $z_1 \in [0.9926, 10.9776]$ , we can represent  $z_1$  as a linear combination of its lower and upper bounds, i.e.,  $z_1 = 0.9926 \times \omega_{11}(x_1(t)) + 10.9776 \times \omega_{12}(x_1(t))$  where  $\omega_{11}(x_1(t)) \geq 0$ ,  $\omega_{12}(x_1(t)) \geq 0$  and  $\omega_{11}(x_1(t)) + \omega_{12}(x_1(t)) = 1$ . This leads to  $\omega_{11}(x_1(t)) = \frac{10.9776 - z_1}{10.9776 - 0.9926}$  and  $\omega_{12}(x_1(t)) = 1 - \omega_{11}(x_1(t))$ . Similarly, we can find  $\omega_{21}(x_1(t))$  and  $\omega_{22}(x_1(t))$  for  $z_2$ ,  $\dots$ , and  $\omega_{51}(x_1(t))$  and  $\omega_{52}(x_1(t))$  for  $z_5$ .

Consequently, the plant can be described by a 32-rule T-S fuzzy model with the rules given below:

**Rule  $i$ :**

IF  $z_1(\mathbf{x}(t))$  is  $M_1^i$ ,  $z_2(\mathbf{x}(t))$  is  $M_2^i$ ,  $z_3(\mathbf{x}(t))$  is  $M_3^i$ ,  $z_4(\mathbf{x}(t))$  is  $M_4^i$ , and  $z_5(\mathbf{x}(t))$  is  $M_5^i$ , THEN  $\dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))$ .

Combining all the fuzzy rules, we have:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{32} w_i(x_1) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) + \mathbf{A}_{di} \mathbf{x}(t - d(t))),$$

where

$$w_i(x_1) = \mu_{M_1^i}(x_1) \times \mu_{M_2^i}(x_1) \times \mu_{M_3^i}(x_1) \times \mu_{M_4^i}(x_1) \times \mu_{M_5^i}(x_1),$$

with

$$\mu_{M_1^i}(x_1) = \omega_{11}(x_1(t)) \text{ for } i = 17, \dots, 32,$$

$$\mu_{M_1^i}(x_1) = \omega_{12}(x_1(t)) \text{ for } i = 1, \dots, 16,$$

$$\mu_{M_2^i}(x_1) = \omega_{21}(x_1(t)) \text{ for } i = 9, \dots, 16, 25, \dots, 32,$$

$$\mu_{M_2^i}(x_1) = \omega_{22}(x_1(t)) \text{ for } i = 1, \dots, 8, 17, \dots, 24,$$

$$\mu_{M_3^i}(x_1) = \omega_{31}(x_1(t)) \text{ for } i = 5, \dots, 8, 13, \dots, 16, 21, \dots, 24, 29, \dots, 32,$$

$$\mu_{M_3^i}(x_1) = \omega_{32}(x_1(t)) \text{ for } i = 1, \dots, 4, 9, \dots, 12, 17, \dots, 20, 25, \dots, 28,$$

$$\mu_{M_4^i}(x_1) = \omega_{41}(x_1(t)) \text{ for } i = 3, 4, 7, 8, 11, 12, 15, 16, 19, 20, 23, 24, 27, 28, 31, 32,$$

$$\mu_{M_4^i}(x_1) = \omega_{42}(x_1(t)) \text{ for } i = 1, 2, 5, 6, 9, 10, 13, 14, 17, 18, 21, 22, 25, 26, 29, 30,$$

$$\mu_{M_5^i}(x_1) = \omega_{51}(x_1(t)) \text{ for } i = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,$$

$$\mu_{M_5^i}(x_1) = \omega_{52}(x_1(t)) \text{ for } i = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31.$$

Here

$$\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \mathbf{A}_4 = \mathbf{A}_5 = \mathbf{A}_6 = \mathbf{A}_7 = \mathbf{A}_8 = \begin{bmatrix} 0 & 10.9776 \\ 1 & 1.9978 \end{bmatrix},$$

$$\mathbf{A}_9 = \mathbf{A}_{10} = \mathbf{A}_{11} = \mathbf{A}_{12} = \mathbf{A}_{13} = \mathbf{A}_{14} = \mathbf{A}_{15} = \mathbf{A}_{16} = \begin{bmatrix} 0 & 10.9776 \\ 1 & -0.0975 \end{bmatrix},$$

$$\mathbf{A}_{17} = \mathbf{A}_{18} = \mathbf{A}_{19} = \mathbf{A}_{20} = \mathbf{A}_{21} = \mathbf{A}_{22} = \mathbf{A}_{23} = \mathbf{A}_{24} = \begin{bmatrix} 0 & 0.9926 \\ 1 & 1.9978 \end{bmatrix},$$

$$\begin{aligned}
\mathbf{A}_{25}=\mathbf{A}_{26}=\mathbf{A}_{27}=\mathbf{A}_{28}=\mathbf{A}_{29}=\mathbf{A}_{30}=\mathbf{A}_{31}=\mathbf{A}_{32} &= \begin{bmatrix} 0 & 0.9926 \\ 1 & -0.0975 \end{bmatrix}, \\
\mathbf{A}_{d1}=\mathbf{A}_{d5}=\mathbf{A}_{d9}=\mathbf{A}_{d13}=\mathbf{A}_{d17}=\mathbf{A}_{d21}=\mathbf{A}_{d25}=\mathbf{A}_{d29} &= \begin{bmatrix} -0.1002 & 0 \\ 0 & -0.1000 \end{bmatrix}, \\
\mathbf{A}_{d2}=\mathbf{A}_{d6}=\mathbf{A}_{d10}=\mathbf{A}_{d14}=\mathbf{A}_{d18}=\mathbf{A}_{d22}=\mathbf{A}_{d26}=\mathbf{A}_{d30} &= \begin{bmatrix} -0.1002 & 0 \\ 0 & -0.1998 \end{bmatrix}, \\
\mathbf{A}_{d3}=\mathbf{A}_{d7}=\mathbf{A}_{d11}=\mathbf{A}_{d15}=\mathbf{A}_{d19}=\mathbf{A}_{d23}=\mathbf{A}_{d27}=\mathbf{A}_{d31} &= \begin{bmatrix} -0.1967 & 0 \\ 0 & -0.1000 \end{bmatrix}, \\
\mathbf{A}_{d4}=\mathbf{A}_{d8}=\mathbf{A}_{d12}=\mathbf{A}_{d16}=\mathbf{A}_{d20}=\mathbf{A}_{d24}=\mathbf{A}_{d28}=\mathbf{A}_{d32} &= \begin{bmatrix} -0.1967 & 0 \\ 0 & -0.1998 \end{bmatrix}, \\
\mathbf{B}_1=\mathbf{B}_2=\mathbf{B}_3=\mathbf{B}_4=\mathbf{B}_9=\mathbf{B}_{10}=\mathbf{B}_{11}=\mathbf{B}_{12} &= \\
\mathbf{B}_{17}=\mathbf{B}_{18}=\mathbf{B}_{19}=\mathbf{B}_{20}=\mathbf{B}_{25}=\mathbf{B}_{26}=\mathbf{B}_{27}=\mathbf{B}_{28} &= \begin{bmatrix} 0.9989 \\ 1 \end{bmatrix}, \\
\mathbf{B}_5=\mathbf{B}_6=\mathbf{B}_7=\mathbf{B}_8=\mathbf{B}_{13}=\mathbf{B}_{14}=\mathbf{B}_{15}=\mathbf{B}_{16} &= \\
\mathbf{B}_{21}=\mathbf{B}_{22}=\mathbf{B}_{23}=\mathbf{B}_{24}=\mathbf{B}_{29}=\mathbf{B}_{30}=\mathbf{B}_{31}=\mathbf{B}_{32} &= \begin{bmatrix} 0.5000 \\ 1 \end{bmatrix}.
\end{aligned}$$

For  $m = 1.0000$ , the maximum delay bound of the polynomial fuzzy model used in Section 5.3 for Case 2 is 0.0795s for Theorem 4 and 0.0175s for Theorem 5. In order to make a fair comparison, we employ Theorem 2 (MFI approach) in Chapter 3 to test the delay bound for the new T-S fuzzy model with other settings unchanged. The test result shows that for delay bound 0.0175s, there is no feasible solution to Theorem 2.

From the above discussion, we can find out that using polynomial fuzzy logic to represent the nonlinear plant would result in comparatively less number of rules compared to the T-S fuzzy logic. In this case, the number of rules is reduced from 32 to 3. This will help simplify the modeling process and lower the computational burden.

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